Reynolds Transport Theorem Applied to Classical Thermodynamics

David Calamas[1], Alan Hewitt[2], John Baker[3], Beth Todd [4]

Abstract – The Reynolds Transport Theorem is often used in undergraduate fluid mechanics courses to transform governing equations from a Lagrangian to an Eulerian coordinate system. As such, it is a useful tool for developing control-volume based expressions for the momentum and conservation of mass equations. Traditional undergraduate thermodynamic texts present the laws of thermodynamics for open systems in a manner that does not directly link them to the original expressions for a closed system. This paper is intended to further student appreciation of the direct connection between the statements of the First and Second Law of Thermodynamics for closed systems and the corresponding statements for open systems. The goal of the paper is to present an alternative approach to teaching thermodynamics that is more closely aligned to instructional methods in fluid mechanics courses.

Keywords: Reynolds Transport Theorem, Thermodynamics, Open System, Closed System, Control Volume

NOMENCLATURE

A	area
B	arbitrary extensive property
b	arbitrary intensive property
CS	control system
CV	control volume
E	energy
g	gravity
h	enthalpy
m	mass
P	pressure
Q	heat transfer
s	entropy
T	temperature
t	time
U	internal energy
V	volume
v	velocity
W	work
θ	methalpy

[1] The University of Alabama, 290 Hardaway Hall (401 7th Avenue), Box 870276 Tuscaloosa, Al, 35487, dmcalamas@crimson.ua.edu

[2] The University of Alabama, 290 Hardaway Hall (401 7th Avenue), Box 870276 Tuscaloosa, Al, 35487, hewit001@crimson.ua.edu

[3] The University of Alabama, 290 Hardaway Hall (401 7th Avenue), Box 870276 Tuscaloosa, Al, 35487, john.baker@eng.ua.edu

[4] The University of Alabama, 290 Hardaway Hall (401 7th Avenue), Box 870276 Tuscaloosa, AL 35487, btodd.eng@gmail.com

v specific volume
ρ density

OPEN AND CLOSED SYSTEMS: A TRADITIONAL APPROACH

Traditional undergraduate thermodynamic texts do not typically utilize the Reynolds Transport Theorem in the derivation of equations governing open systems. Three recent undergraduate level thermodynamic texts have been reviewed to form an opinion on the traditional approach to closed and open systems [1-3]. The concept of open systems, and subsequently control volumes, is traditionally introduced with the First Law. Mathematically, the First Law of Thermodynamics is expressed as

$$\delta\dot{Q} - \delta\dot{W} = \frac{DE}{Dt} \tag{1}$$

where $\delta\dot{Q}$ is the differential time rate of change of heat transfer across system boundaries, $\delta\dot{W}$ is the differential time rate of change of work across system boundaries, and E is the energy contained within the system. It should be noted the total energy contained within the system will be limited to contributions from internal, kinetic, and potential energies. In the above definition, the term system refers to a fixed mass that is moving through space. As such, the derivative appearing on the right hand side of the equation is a Lagrangian reference frame derivative. The introduction of open systems traditionally precedes the First Law analysis of closed systems. By definition, a closed system is defined as a system where energy transfer across system boundaries is possible but where no mass transfer may occur across the boundaries of the system. Classical examples of closed systems include piston-cylinder devices and sealed rigid tanks. For a closed system we are interested in a control mass as it moves through space. For steady-state processes, the First Law of Thermodynamics may be recast as

$$Q - W = \Delta E \tag{2}$$

where ΔE is the change in total energy for the system from the initial state to the final state. Defining a stationary system as one with negligible potential and kinetic energy changes further simplifies the First Law of Thermodynamics as

$$Q - W = \Delta U \tag{3}$$

where ΔU is the change in the total internal energy for the system from the initial to the final state. An open system is defined as a system where both energy and mass can be transferred across system boundaries. Classical examples of open systems include turbines, compressors, and heat exchangers. For an open system we are interested in a control volume which may be exchanging mass with the surroundings. Most thermodynamics text books apply an energy balance to a control volume to develop the following form of the First Law of Thermodynamics for open systems

$$\delta\dot{Q} - \delta\dot{W} + \sum_{in}\dot{m}\theta - \sum_{out}\dot{m}\theta = \frac{\partial E}{\partial t} \tag{4}$$

While an energy balance applied to a control volume may provide the student with a "physical" appreciation of the transfer of energy and mass across the boundaries of the open system, it does not automatically lead to a fundamental appreciation of the difference between Eulerian and Lagrangian reference frames. As with Newton's Second Law of Motion, the First Law of Thermodynamics is written for a fixed mass, i.e. a control mass. In a typical undergraduate fluid mechanics course, the Reynolds Transport Theorem is used to develop expressions for Newton's Second Law of Motion in Eulerian reference frames, i.e. with respect to a control volume exchanging matter with its surroundings. The same approach can be taken with respect to developing expressions for the First and Second Laws of Thermodynamics for control volumes.

REYNOLDS TRANSPORT THEOREM: FLUID MECHANICS

The concept of open system analysis as presented in the reviewed thermodynamics texts [1-3] will be compared to open system analysis as presented in several undergraduate fluid mechanics texts [4-6]. There is a fundamental difference in the introduction of open system analysis between the reviewed thermodynamics and fluid mechanics texts. The reviewed fluid mechanics texts do not incorporate the First Law of Thermodynamics in the introduction of open systems. The concept of open systems is presented in a more thorough and detailed manner in undergraduate fluid mechanics texts as it is introduced in the derivation of Reynolds Transport Theorem. Unfortunately, the Reynolds Transport Theorem is not widely used for developing statements of the First and Second Laws of

Thermodynamics for open systems in traditional undergraduate thermodynamics texts. Similarly, unlike the thermodynamics texts, mathematical details are not spared in the fluid mechanics texts. While mathematically rigorous, the use of Reynolds Transport Theorem has the potential to help students physically and conceptually relate closed and open systems. Anecdotal evidence suggests that undergraduate mechanical engineering students often do not fully appreciate that there is a direct connection between the statement of the First Law of Thermodynamics for a closed system and the corresponding statement for an open system. The aforementioned Reynolds Transport Theorem as presented in the reviewed fluid mechanics texts [4-6] is

$$\frac{DB}{Dt} = \frac{\partial}{\partial t}\iiint_{CV} b\rho dV + \oiint_{CS} b\rho\bar{v}\cdot d\bar{A} \quad (5)$$

The left side of Eq. 5 is the Lagrangian form and represents the rate of change of the arbitrary property, *b*, of the system. The right side is the Eulerian form and represents the change of the arbitrary property in the control volume and the flux of the property through the control surface.

REYNOLDS TRANSPORT THEOREM: APPLIED TO FIRST LAW OF THERMODYNAMICS

The Reynolds Transport Theorem can be applied to properties to derive open system formulations. It will be demonstrated that Reynolds Transport Theorem can be applied to the closed form of the First Law of Thermodynamics to yield the open system form of the First Law of Thermodynamics. The First Law for a closed system is

$$\delta\dot{Q} - \delta\dot{W} = \frac{DE}{Dt} \quad (6)$$

The energy, an extensive property, contained within a system is

$$E = U + m\frac{v^2}{2} + mgz \quad (7)$$

The specific energy or energy per unit mass, of a system is

$$e = u + \frac{v^2}{2} + gz \quad (8)$$

Equating the total energy of the system, *E*, with the extensive property, *B*, and the specific energy of the system, *e*, with the intensive property, *b*, yields the following when substituted into the Reynolds Transport Theorem

$$\frac{DE}{Dt} = \frac{\partial}{\partial t}\iiint_{CV} e\rho dV + \oiint_{CS} e\rho\vec{v}\cdot d\vec{A} \quad (9)$$

The First Law of Thermodynamics now may be written as

$$\delta\dot{Q} - \delta\dot{W} = \frac{\partial}{\partial t}\iiint_{CV} e\rho dV + \oiint_{CS} e\rho\vec{v}\cdot d\vec{A} \quad (10)$$

and for a homogeneous material as

$$\delta\dot{Q} - \delta\dot{W} = \frac{\partial e\rho V}{\partial t} + \oiint_{CS} e\rho\vec{v}\cdot d\vec{A} \quad (11)$$

The integral represent the net flux rate of the extensive property, in this case energy, through the control surface. Assuming uniform flow, i.e. the properties across a given cross sectional area are uniform; the above equation may be rewritten as

$$\delta\dot{Q} - \delta\dot{W} = \frac{\partial E}{\partial t} + \sum_{out} e\rho vA - \sum_{in} e\rho vA \quad (12)$$

using the definition of the mass flow rate

$$\dot{m} = \rho vA \quad (13)$$

yields

$$\delta\dot{Q} - \delta\dot{W} + \sum_{in}\dot{m}e - \sum_{out}\dot{m}e = \frac{\partial E}{\partial t} \quad (14)$$

By distinguishing flow work from other types of work the First Law of Thermodynamics may be written as

$$\delta\dot{Q} - \delta\dot{W} - \delta\dot{W}_{flow} + \sum_{in}\dot{m}e - \sum_{out}\dot{m}e = \frac{\partial E}{\partial t} \tag{15}$$

Flow work is the work required to move mass in and/or out of the system and may be written as

$$w_{flow} = Pv \tag{16}$$

and thus

$$\delta\dot{W}_{flow} = \dot{m}\delta w_{flow} = \dot{m}\delta\left(\vec{P}\cdot\vec{v}\right) = (\dot{m}Pv)_{out} - (\dot{m}Pv)_{in} \tag{17}$$

Equation 15 can now be written as

$$\delta\dot{Q} - \delta\dot{W} + (\dot{m}Pv)_{in} - (\dot{m}Pv)_{out} + \sum_{in}\dot{m}e - \sum_{out}\dot{m}e = \frac{\partial E}{\partial t} \tag{18}$$

or,

$$\delta\dot{Q} - \delta\dot{W} + \sum_{in}\dot{m}\left(Pv + u + \frac{v^2}{2} + gz\right) - \sum_{out}\dot{m}\left(Pv + u + \frac{v^2}{2} + gz\right) = \frac{\partial E}{\partial t} \tag{19}$$

Using the definition of enthalpy

$$h = u + Pv \tag{20}$$

and the definition of methalpy

$$\theta = h + m\frac{v^2}{2} + mgz \tag{21}$$

Results in the First Law of Thermodynamics for an open system

$$\delta\dot{Q} - \delta\dot{W} + \sum_{in}\dot{m}\theta - \sum_{out}\dot{m}\theta = \frac{dE}{dt} \tag{22}$$

The steps in this derivation show that the Reynolds Transport Theorem can be successfully used to represent the First Law of Thermodynamics for an open system.

REYNOLDS TRANSPORT THEOREM: APPLIED TO THE ENTROPY EQUATION

As with the First Law, it will be demonstrated that Reynolds Transport Theorem can be applied to yield the open system form of the Second Law of Thermodynamics. The Second Law of Thermodynamics defines two thermodynamic properties, absolute temperature and entropy. These properties are related by the Clasius inequality

$$\frac{DS}{Dt} \geq \frac{\delta\dot{Q}}{T} \tag{23}$$

The integral form of which is found through the application of Reynolds Transport Theorem

$$\frac{\partial}{\partial t}\iiint_{CV} s\rho dV + \oiint_{CS} s\rho\vec{v}\cdot d\vec{A} \geq \oiint_{CS}\left(\frac{-\dot{Q}}{T}\right)dS \tag{24}$$

In accordance with the increase of entropy principle an entropy generation term, S_{gen}, will be introduced into Eq. 24 to remove the inequality. Therefore, the Second Law of Thermodynamics may be written as

$$\frac{\partial}{\partial t}\iiint_{CV} s\rho dV + \oiint_{CS} s\rho\vec{v}\cdot d\vec{A} = \oiint_{CS}\left(\frac{-\dot{Q}}{T}\right)dA + \dot{S}_{gen} \tag{25}$$

and for a homogeneous material as

$$\frac{\partial s\rho V}{\partial t} + \oiint_{CS} s\rho\vec{v}\cdot d\vec{A} = \oiint_{CS}\left(\frac{-\dot{Q}}{T}\right)dA + \dot{S}_{gen} \tag{26}$$

Assuming uniform properties across a given cross sectional area and uniform heat addition over the control surface; the above equation may be rewritten as

$$\frac{\partial S}{\partial t} + \sum_{out} s\rho vA - \sum_{in} s\rho vA = -\sum\left(\frac{\dot{Q}}{T}\right) + \dot{S}_{gen} \tag{27}$$

Using the definition of the mass flow rate becomes

$$\frac{\partial S}{\partial t} + \sum_{out} \dot{m}s - \sum_{in} \dot{m}s = -\sum \left(\frac{\dot{Q}}{T}\right) + \dot{S}_{gen} \quad (28)$$

or

$$\sum \left(\frac{\dot{Q}}{T}\right) + \sum_{in} \dot{m}s - \sum_{out} \dot{m}s + \dot{S}_{gen} = \frac{\partial S}{\partial t} \quad (29)$$

The steps in this derivation show that the Reynolds Transport Theorem can, like the First Law, be successfully used to represent the Second Law of Thermodynamics for an open system.

CONCLUSIONS

The Reynolds Transport Theorem, while mathematically rigorous, easily shows the relationship between closed and open systems in thermodynamics as shown previously in this work. The use of the Reynolds Transport Theorem as applied to the First and Second Law of Thermodynamics in undergraduate thermodynamics would help to unify the use of Reynolds Transport Theorem as applied to the continuity and momentum equations in fluid mechanics.

ACKNOWLEDGEMENTS

David Calamas is funded by the University of Alabama GK-12 Program. The GK-12 Program is supported by the National Science Foundation under grant 0742504. Alan Hewitt is funded by the Department of Education through a Graduate Assistance in Areas of National Need (GAANN) fellowship.

REFERENCES

[1] Borgnakke, Claus and Richard Sonntag, *Fundamentals of Thermodynamics 7th Edition*, John Wiley and Sons, Inc., New Jersey, 2009, pg. 180-187.

[2] Cengel, Yunus and Michael Boles, *Thermodynamics: An Engineering Approach 6th Edition*, McGraw-Hill, New York, 2008, pg. 220-233.

[3] Moran, Michael and Howard Shapiro, *Fundamentals of Engineering Thermodynamics 5th Edition*, John Wiley and Sons, Inc., New Jersey, 2004, pg. 131-143.

[4] Crowe, Clayton , Elger, Donald and John Robertson, *Engineering Fluid Mechanics 8th Edition*, John Wiley and Sons, Inc., New Jersey, 2005, pg. 147-154.

[5] Munson, Bruce, Young, Donald and Theodore Okiishi, *Fundamentals of Fluid Mechanics*, John Wiley and Sons, Inc., New Jersey, 1990, pg. 223-237.

[6] White, Frank, *Fluid Mechanics 4th Edition*, McGraw-Hill, New York, 1999, pg. 129-141.

David Calamas

David Calamas is a graduate student in the Department of Mechanical Engineering at The University of Alabama. David Calamas is funded by the GK-12 program supported by the National Science Foundation under grant 0742504. His research interests include thermal transport in systems with biologically-inspired geometries.

Alan Hewitt

Alan Hewitt is a graduate student in the Department of Mechanical Engineering at The University of Alabama. Alan Hewitt is funded by a Department of Education GAANN Fellowship. His research interests include computational modeling of heat transfer and fluid flow in a stirling engine. He has taught two semesters of undergraduate thermodynamics.

John Baker

Dr. John Baker is a professor in the Department of Mechanical Engineering at The University of Alabama. His research interests include heat transfer, fluid dynamics and thermodynamics.

Beth Todd

Beth Todd is an associate professor in the Department of Mechanical Engineering at The University of Alabama. Her research interests include engineering education.