# Mathematical Skills in Electronics Engineering Technology Curriculum

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**Abstract** – As described by the National Society of Professional Engineers, engineering programs are built on a foundation of complex mathematics and science courses; engineering technology programs provide students introductory mathematics and science courses, and only a qualitative introduction to engineering fundamentals. Engineering technology instructors as well as engineering technologists often focus on the applied and practical application of engineering principles, and thus lack foundation of complex mathematics and science courses. During the six years of engineering technology education experience, the author believes that introduction of higher level mathematics skills into engineering technology curriculum benefits the graduates in product design, testing, development, and systems development.

In this paper, extensive examples in circuit analysis, analog and digital communications systems, and electrical power are adopted to illustrate how higher level mathematics are incorporated into engineering technology curriculum. How to choose appropriate breadth and depth of the mathematics skills is discussed. Students' response through survey questionnaire is provided. Results show that well-planned integration of mathematics and techniques is efficient in creating a learning environment that nurtures critical thinking skills and development of technology expertise.

*Keywords:* Electronics engineering technology, Mathematical skills, Science, technology, engineering, and math (STEM).

# INTRODUCTION

The importance of mathematics will never be overemphasized in engineering and engineering technology education. However, inadequate mathematical skills have presented a widespread problem throughout engineering and technology fields; major reports [1–5] have been released by highly respected U.S. academic, scientific, and business organizations on the need to improve mathematics education [6]. The National Academies reported in 2010 that among 29 wealthy countries, the United States ranked 27<sup>th</sup> in the proportion of college students with degrees in science and engineering; among developed countries, the United States ranks 31<sup>st</sup> in math; American 12<sup>th</sup> graders were near the bottom of students from 20 nations assessed in advanced math and physics [7]. The National Academies has warned that the US will continue to lose ground to foreign economic rivals.

In 2001, Boston-based MathSoft Education and the American Society for Engineering Education released a study result stating that 40 percent of U.S. high school students lack the fundamental mathematics skills required to complete college engineering degrees [8]; ten years later, Microsoft Corporation announced the findings of national surveys conducted online by Harris Interactive. The result was still shocking. Only 20 percent of college students agreed that their high-school math courses prepared them well enough for their college courses [9].

Mathematical skills have long been considered a merit among engineers and technicians. As a highly educated and skilled labor force drives innovation and production, it looks like there is no doubt that math and science creates more jobs [7]. However, the facts seem to have deviated from perspectives. While half of parents surveyed by Microsoft claimed they would like to see their children pursue a career in an STEM field, but only 24 percent are

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willing to pay extra money to help them succeed in math and science courses. How to improve the skills in math and science still remains a compelling task in higher education.

There has been a lot of practice in recent years aiming to bridge the gap between the future requirements of mathematical skills and the reality. Special curriculum [10] has been designed to target core engineering skills and concepts that should be addressed at the high school level in order to better prepare students to pursue engineering and technology related careers. Program [11] has been implemented, where a 30-credit hour module was designed to provide students with mathematical understanding and confidence prior to commencing a degree in an engineering and technology capacity; and enhancing skills through interest group [12]. All practices have shown promise. Most of these practices need more resources beyond degree requirements. As a decreasing trend can be seen recently in the total credit hours required for a degree, it is not always practical to offer separate programs or even independent courses for the purpose of improving student mathematics skills. An alternative option is to increase the communication between mathematics and engineering faculty and to develop joint resources for problematic areas [13]. This is essential, because if interactions between math and engineering technology faculty are absent, when theories in math are taught, the instructor will not be able to mention its particular applications into engineering and technology, and the students have no ideas how to apply the theories into the engineering and technical fields; similarly, when an engineering, especially engineering technology course is taught, the students or sometimes even the instructor lack an advanced, analytical point of view. Although both mathematics and engineering technology faculties should be involved in the process, it is the author's opinion that engineering, especially technology instructors should take more responsibility. The reason is that students in traditional mathematics classes may come from wide range of majors, and some of them might not be engineering and technology related. While in engineering or technology classes, the required mathematics skills are relatively more focalized, and thus it is more practical and productive for the engineering and technology faculty to apply fundamental mathematics skills into engineering and technology fields with the consulting from mathematics faculty if necessary.

The following sections describe how the author as a technology instructor integrates math skills into typical electronics engineering technology curriculum. General approach and typical topics are discussed followed by student survey results and discussions.

## **APPROACH**

The Electronics Engineering Technology (EET) Program in University of Southern Mississippi is taking actions to promote its academic standing by offering more high-level engineering and technology courses. Several electrical engineering courses, such as signal processing, signals and systems, electromagnetic fields and waves, antenna analysis and design, alternative energy have been offered as technical electives in addition to its traditional required course including circuit analysis, electrical power, and analog and digital communications. The author has taken every opportunity to integrate mathematical skills into the technology curriculum. It is obviously that many math skills are needed in an individual course, and each skill may be essential in multiple courses. The author tried to emphasize the most necessary math skills in the most appreciate courses, for example, complex number operations in electric power and trigonometry, FD and TD representations in analog and digital communications. However, quick examples in other courses were also used to broaden the application of the math skills. The author believes that the liaison across courses within an integrated curriculum promotes the students' interest yet does not take much time. However, compared with traditional practice, extra resources, although not much, are needed from the instructor to secure a success.

## Identify the problem

The faculty in engineering technology program discuss on a regular basis to determine the math topics that are essential and need improvements based on experience from previous years. At the beginning of the semester, diagnostic was performed in each involved course. Problems from database were used in the quiz to determine how solid the students' math skills are and to test the ability the students have to apply math skills into solving engineering technology problems.

## Major math skills

The diagnostic identified the following math skills that need improvements from the above mentioned courses:

- a. complex numbers; vector analysis
- b. trigonometry; linear systems
- c. frequency and time domain representations; Fourier transforms
- d. differentiation; Taylor expansion
- e. differential equations
- f. linear algebra: matrix multiplication
- g. special functions: rectangle pulse train, Bessel, Sinc function.

These topics do not cover all necessary mathematical skills, but do create a math foundation for the entire electronics engineering technology curriculum.

# **CORE MATH SKILLS IN EET PROGRAM**

## **Complex number operation**

Complex numbers and operations are introduced very early in the math sequence. A typical use of complex numbers and operations is in the discussion of concepts of active power, reactive power, and apparent power. When AC power are introduced in two-year colleges, active power P, reactive power Q, and apparent power S are usually defined as

$$P = VI\cos\theta, Q = VI\sin\theta, \text{ and } S = VI, \tag{1}$$

where V, I, and  $\theta$  are effective voltage values, effective current value, and the phase of voltage relative to current. This notation has limit when dealing with complex circuits, where complex power is more convenient to use. The complex power is defined as  $S_c = VI^*$ , where \* stands for complex conjugate. In this definition, the real part of the complex power is the active power; the imaginary part is the reactive power, and the absolute value is the apparent power.

#### Trigonometry

Trigonometry is a prerequisite course for Calculus I. We found that trigonometry is important in analog communications. For example, before the concept of modulation and demodulation is introduced, we always review the trigonometric identities, sum as product-to-sum formulas. For example, in trigonometry,

$$\cos\alpha\,\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)].$$
(2)

If two sinusoidal signals with frequencies  $\omega_1$  and  $\omega_2$  are multiplied together, then

$$\cos\omega_1 t \, \cos\omega_2 t = \frac{1}{2} [\cos(\omega_1 + \omega_2) t + \cos(\omega_1 - \omega_2) t]. \tag{3}$$

This is the concept of modulation and demodulation. As there are two new frequencies generated in the process, it is concluded that modulator and demodulator are actually multipliers and both are nonlinear devices. On the contrary, the sum-to-product formula

$$\cos\omega_1 t + \cos\omega_2 t = 2\left[\cos\frac{\omega_1 + \omega_2}{2}t + \cos\frac{\omega_1 - \omega_2}{2}t\right]$$
(4)

represents the concept of musical beat.

# Linear systems

Linear systems linear systems find important applications in automatic control theory signal processing, and telecommunications. From technical view, there are no new frequencies generated. Exponentials are eigenfunctions of linear systems. This means, for an exponential input, the output is the input multiplied by a constant that depends on the system response at that frequency, thus exponential signal  $e^{j\omega t}$ , instead of sinusoidal signal, is often used to analyze linear systems.

### Time domain and frequency domain representation, Fourier transform

Time domain (TD) signals are first introduced in AC circuit analysis where time domain signals can be observed vividly on oscilloscopes. Frequency domain (FD) representation is usually introduced later in frequency response analysis. An FD signal illustrates the frequency components of the TD signal and can be easily observed on a spectrum analyzer. A given signal can be converted between the time and frequency domains with a pair of mathematical operators called a Fourier transform and inverse Fourier transform. Fourier transform decomposes a TD signal into the sum of a series of sine wave frequency components. The inverse Fourier transform converts the frequency domain signal back to a time function. An excellent application of these concepts into engineering technology is modulation and demodulation in analog communications systems. The multiplication of two time domain baseband signal and carrier signal results in two frequency shifts: upper band frequency and lower band frequency. The combination of TD and FD representations makes technology students understand modulation and demodulation and demodulation frequency shifts: upper band frequency and lower band frequency.

#### Differentiation, integration, and Taylor expansion

Differentiation and integration are used in circuit analysis to analyze some devices and systems, such as filters. For example, a differentiator is a high pass filter while an integrator is a low pass filter. This is because, for a input exponential signal  $x(t) = e^{j\omega t}$ , the differentiation is

$$y(t) = \frac{\mathrm{d}x(t)}{\mathrm{d}t} = j \,\omega \, x(t), \tag{5}$$

and the integration is

$$y(t) = \int x(t) = \frac{1}{j\omega} x(t).$$
 (6)

Obviously, the output of the differentiation is proportional to frequency and thus a high pass filter; while the output of the integrator is inversely proportional to frequency and thus a low pass filter.

The Taylor series of a signal f(t) that is infinitely differentiable in a neighborhood of  $t_0$  is the power series

$$f(t) = f(t_0) + \frac{f'(t_0)}{1!} (t - t_0) + \frac{f''(t_0)}{2!} (t - t_0)^2 + \frac{f'''(t_0)}{3!} (t - t_0)^3 + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(t - t_0)}{n!} (t - t_0)^n.$$
(7)

By using Taylor expansion, the angle modulated time domain signal can be decomposed into a series of

$$\varphi(t) = A[\cos\omega_c t - k_f a(t) \sin\omega_c t - \frac{k_f^2}{2!} a^2(t) \cos\omega_c t + \frac{k_f^3}{3!} a^3(t) \sin\omega_c t + \cdots],$$
(8)

where  $\omega_c$  is the carrier frequency, a(t) is related to baseband signal. This means, the spectrum of the angled modulated signal consists of un-modulated carrier plus the spectra of multiples of the baseband signals centered at the carrier frequency.

#### Differentiations

Solving differential equation is a challenging task for engineering technology students. A relatively simple approach is demonstrated below. Assume that the first order electrical system to be solved is

$$L\frac{di(t)}{dt} + Ri(t) = v(t), \tag{9}$$

and the input forcing signal is sinusoidal  $v(t) = v_0 \cos \omega t$ . Write the current i(t) in the form of an exponential  $i(t) = i_0 e^{j\omega t}$ , then

$$\frac{\mathrm{di}(t)}{\mathrm{dt}} = j\omega \,i_0 \mathrm{e}^{j\omega \mathrm{t}}.\tag{10}$$

By plugging the derivative into the original equation (9),  $i_0$  can be easily determined. The final solution is the real part of the current i(t).

#### **Matrix multiplication**

Matrix multiplication is important in digital communications. It is helpful in understanding the linear block code design process. Assuming in (n, k) Hamming code, the codes are  $c_1, c_2, c_3, \dots, c_n$ , and the data are  $d_1, d_2, d_3, \dots, d_k$ . In general cases, the codes are linear combinations of the data bits. One example is

$$\begin{cases} c_1 = d_1 \\ c_2 = d_2 \\ \dots \\ c_k = d_k \\ c_{k+1} = h_{11}d_1 + h_{12}d_2 + \dots + h_{1k}d_k \\ \dots \\ c_n = h_{m1}d_1 + h_{m2}d_2 + \dots + h_{mk}d_k \end{cases}$$
(11)

where  $h_{ij}$  are 0 or 1, and all operations are binary. The equations can be written into matrix form [c] = [d][G], where [c], [d], and [G] are code, data, and generator matrices, respectively. The generator matrix is

$$[G] = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & h_{11} & \dots & h_{m1} \\ 0 & 1 & 0 & \dots & 0 & h_{12} & \dots & h_{m2} \\ & & & & & \\ 0 & 0 & 0 & \dots & 1 & h_{1k} & \dots & h_{mk} \end{bmatrix}.$$
 (12)

The result of a matrix  $d_{N \times k}$  multiplied by a matrix  $G_{k \times n}$  from the right hand side equals the linear combination of the columns of  $d_{N \times k}$ , while the coefficients of the combination are determined by matrix  $G_{k \times n}$ . This concept gives students a quick calculation of the codes as well as a better understanding of the design process.

# **ASSESSMENT RESULTS**

So far the practice has been assessed by one indirect method (student surveys) and one direct method (diagnostics by the instructors). For each involved course, there are two student surveys conducted at the beginning and at the end of the semester. The students are asked to answer a series of questions to evaluate the math skills. Results are tabulated for a three-year period from fall semester, 2008 for courses EET315: analog communications systems, CET316: digital communications; and EET461: electric power generation, transmission, and distribution. The total number of feedbacks is 73. Responses from students who withdraw from classes at later times have been removed from the statistics, although the exclusions are rare. Each question has five choice that are scored one to five, respectively, standing for "strongly disagree", "disagree", "neutral", "agree", and "strongly agree". The typical questions are:

- 1. I have enough skills in complex numbers and operation.
- 2. I have enough skills in trigonometry.
- 3. I have enough skills in linear systems.
- 4. I have enough skills in frequency and domain representation of signals.
- 5. I have enough skills in Fourier transform.
- 6. I have enough skills in differentiation and integration.
- 7. I have enough skills in Taylor expansion.
- 8. I have enough skills in differential equations.
- 9. I have enough skills in matrix multiplication.
- 10. I have enough skills in some special functions, such as impulse function.

Survey question from 1 to 10 examine the math skills discussed in this paper. The average scores for each of these questions answered at the beginning and at the end of semester are plotted in comparison in Figure 1. From the bar plot we see that for each of the ten mathematical skills, there is obvious improvement through the study of the

course. The most prominent improvement occurs in complex numbers and operations, frequency domain representations, and matrix multiplication. This is because these math skills have been used a lot in electrical power, analog and digital communications.



Figure 1: Average survey scores at the beginning and end of semester

For assessment purpose, a database of problems of similar difficulty level is maintained. Any quiz problem before or after the semester will be extracted from the database. The students will be told that the pre-quiz results do not affect their class performance and there is no indication that another quiz at the end of the semester exists.

The 10 quiz problems test student skills in the following area:

- 1. complex numbers and operation,
- 2. trigonometry,
- 3. linear systems,
- 4. frequency and domain representation of signals,
- 5. Fourier transform,
- 6. differentiation and integration,
- 7. Taylor expansion,
- 8. differential equations,
- 9. matrix multiplication,
- 10. some special functions, such as impulse function.

The average scores for the diagnostic quizzes right after each survey are plotted in comparison in Figure 2. We see that in the direct assessment, students have also shown improvements in each of the math skills tested. A close examination of Figure 1 and Figure 2 shows that the students tend to overestimate their skills in the survey, both before and after the courses.



Figure 2: Average quiz scores at the beginning and end of semester

# **CONCLUSIONS AND DISCUSSIONS**

In our previous practice [14] we proposed a new course among engineering technology students aiming to enhance the math and science skills. The course has been offered for three semesters; however, we found that students lack motivation to take the course, one reason being that it is not a required course in the curriculum. Instead, an alternative practice [15] has obtained some satisfactory results. In this paper, we described our effort to apply the pedagogy across the electronics engineering technology curriculum. Evaluation results have been provided at the course level. It is concluded that the mathematics skills of students, especially of transfer students, have been improved, which in turn, their academic performance in electronics engineering technology as well as future learning skills will be improved.

There are other issues that we are facing now and in the future. First, more direct methods are necessary to be adopted in the assessment process; second, the assessment of the effect on the student performance at the program level is a meaningful task; third, a systematic method to determine the scope of math skills needs to be implemented on the present foundation; and at last, communications and cooperation with mathematics instructors are encouraged. These tasks remain in progress.

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