Use of MathCad and Excel to Enhance the Study of Psychrometric Processes for Buildings in an Air Conditioning Course

Rogelio Luck[1] and Pedro J. Mago[2]

Abstract – The use of computational tools such as Mathcad or Microsoft Excel to increase the level of conceptual understanding of psychrometric processes in buildings in an air conditioning technical elective course is explored. By utilizing Mathcad and/or Excel students can be taught to create a set of psychrometric related functions that can greatly diminish the number of calculations required to tackle design-oriented problems. By doing this, two levels of conceptual understanding are enhanced. First students gain an appreciation of the basic psychrometric processes for buildings when creating specific functions in Mathcad/Excel, and second, students gain the ability to create automated worksheets for specific applications allowing them to see immediate results to variations in input design conditions as well as on different parameters. Feedback from students as well as class instructors demonstrates that the use of these tools enhance the student experience in an air conditioning class.

Keywords: MathCad, Air Conditioning, Excel, Psychrometrics.

INTRODUCTION

The use of air conditioning is indispensable in the southeast of the US due to the high temperatures and humidity. For this reason, mechanical engineering students typically demonstrate high levels of interest in the air conditioning topic. At Mississippi State University, ME 4563 Air Conditioning, is taught as a technical elective/beginning graduate course in the Mechanical Engineering program. The course includes: psychrometric fundamentals, psychrometric processes for buildings, thermal comfort, thermal loads, duct design, and cooling equipment. However, the topics of psychrometric fundamentals and psychrometric processes for buildings are emphasized at the beginning of the class since this information is crucial for the rest of the semester and the final design project. McClain and Smitherman [1] developed MathCAD functions to evaluate the thermodynamic properties of moist air as well as ammonia, propane, and Refrigerant 22. This paper explores the use of Mathcad and Excel to enhance the classroom presentation and to increase the level of understanding of psychrometric processes in buildings in an air conditioning technical elective course. Typical psychrometric processes in buildings are covered using psychrometric formulations and the psychrometric chart and the computations, i.e., the “quantitative” experience, does not extend beyond routine calculations for specified design conditions. By utilizing Mathcad and or Excel to guide the students to develop a Mathcad or Excel tool with an interface in which they only have to input the design conditions and other parameters of interest, the student experience has been extended significantly. In particular, students can quickly see the effect of the variation of different parameters and use this information for system selection; which represents a significant and useful enhancement to the course. Some of the advantages of Mathcad over Excel for the proposed problem include the clarity of notation (it is easy to read because the equations have the same form, nomenclature, and overall appearance as in the analytical derivations) and the ability to automatically perform unit analysis. However, students are usually familiar with the use of Excel so there is a shallow learning curve and Excel is well adapted for handling data files such as property data files for different substances. Excel also provides a more comprehensive and flexible tool for 2D plotting. Discussion and examples of the use of Mathcad and Excel in the air conditioning course are presented in the following sections.

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ANALYSIS

Figure 1 illustrates a schematic of the cooling and dehumidifying system for a typical air conditioning system. For air conditioning during summer time, the air must be supplied at a low temperature and humidity so as to absorb the total heat and humidity load of the space. As the air flows through the space, it is heated and humidified. As seen in Fig. 1, outdoor air is mixed with the return air and sent to the cooling and dehumidifying equipment, where it is cooled and dehumidified and supplied to the conditioned space again. The outdoor air requirements depends on the type of building and the occupants. To be able to analyze the various processes involved in an air conditioning for a building, the properties of the moist air at the different states must be determined. These properties include: the humidity ratio, relative humidity, specific enthalpy, and specific volume.

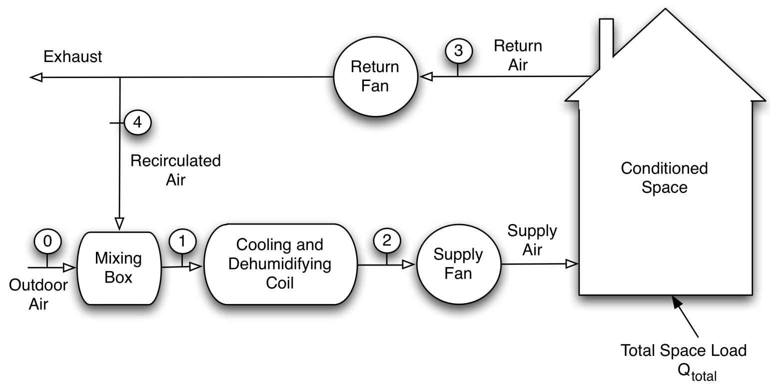

Figure 1. Schematic of cooling and dehumidifying System

The properties of moist air at the different states can be determined as follow [2]:

a. The saturation pressure (Pws):

$$Pws = exp\left(\frac{C_1}{T} + C_2 + C_3T + C_4T^2 + C_5T^3 + C_6\ln(T)\right) \quad (1)$$

where

$C_1 = -5{,}800.2206$ $\quad C_2 = 1.3914993$ $\quad C_3 - 0.048640239$

$C_4 = 4.1764769\ x\ 10^{-5}$ $\quad C_5 = -1.4452093\ x\ 10^{-8}$ $\quad C_6 = 6.5459673$

In Eq. (1) *Pws* is in kPa and *T* in K.

b. The relative humidity (∅):

$$\emptyset = \frac{Pw}{Pws} \quad (2)$$

where *Pw* is the partial pressure of water vapor.

c. The humidity ratio (W):

$$W = 0.62198\frac{Pw}{P-Pw} \quad (3)$$

where *P* is the total pressure.

d. The specific volume (v):

$$v = 0.2871T\frac{(1+1.6078W)}{P} \quad (4)$$

e. The specific enthalpy:

$h = 1.006t + W(2,501 + 1.86t)$ (5)

In Eq. (5) h is in kJ/kg and t in °C.

Methodology

During the air conditioning analysis, the most important outcomes are the capacity of the cooling and dehumidifying equipment and the quantity of supplied air, since these two parameters are used for the equipment selection. This section presents a methodology to determine these two quantities using the psychrometric formulations. Although the methodology can be performed using the psychrometric chart, using computational tools such as MathCad or Excel, enhances the learning process and allows the students to quickly determine the effect of different parameters on the air conditioning performance. The steps that are needed to determine the capacity of the cooling and dehumidifying equipment are as follows:

1. With the total heat load of the building, $\dot{Q}_{total}$, and the sensible heat, $\dot{Q}_{sensible}$, the room sensible heat ratio can be determined as:

$SHR_R = \frac{\dot{Q}_{sensible}}{\dot{Q}_{total}}$ (6)

2. Find properties of State 3 using T_3 and $\emptyset_3$ → Pws_3, Pw_3, W_3, h_3 using Equations (1-5)

3. The temperature of the supply air can be determined assuming a temperature difference between T_2 and T_3 as follows:

$\Delta T = T_3 - T_2$ (7)

This temperature difference is usually assumed to be around 10°C or less [3].

4. With the SHR_R and T_2 the enthalpy of State 2 can be determined as:

$h_2 = h_3 - \frac{q_{sensible}}{SHR_R}$ (8)

5. Find the rest of the properties of State 2 using h_2 and T_2 → Pws_2, Pw_2, W_2, v_2 using Equations (1-5)

6. The mass flow rate ($\dot{m}_{a,2}$) and volumetric flow rate ($\dot{V}_{a,2}$) of air required are, respectively:

$\dot{m}_{a,2} = \frac{Q_{total}}{h_3 - h_2}$ (9)

$\dot{V}_{a,2} = \dot{m}_{a,2} v_2$ (10)

7. Find the properties of State 0 using with T_0 and $\emptyset_0$ → Pws_0, Pw_0, W_0, h_0, and v_0 using Equations (1-5)

8. The mass flow rate of fresh air ($\dot{m}_{a,0}$):

$\dot{m}_{a,0} = \frac{\dot{V}_{a,0}}{v_0}$ (11)

9. The amount of recirculated air ($\dot{m}_{a,4}$) can be obtained by doing a mass balance on the mixing section:

$$\dot{m}_{a,4} = \dot{m}_{a,2} - \dot{m}_{a,0} \quad (12)$$

10. To identify State 1, the enthalpy of State 1 can be obtained as

$$h_1 = \frac{h_0\dot{m}_{a,0} + h_3\dot{m}_{a,4}}{\dot{m}_{a,2}} \quad (13)$$

and T_1 can be obtained from a similar triangle theory as:

$$\frac{T_1 - T_3}{T_0 - T_3} = \frac{h_1 - h_3}{h_0 - h_3} \quad (14)$$

11. Find the rest of the properties of State 1 using with T_1 and $h_1 \rightarrow Pws_1, Pw_1, W_1$ using Equations (1-5)

12. The capacity of the cooling and dehumidifying equipment ($\dot{Q}_{coil}$) is:

$$\dot{Q}_{coil} = \dot{m}_{a,2}(h_1 - h_2) \quad (15)$$

12. The coil sensible heat ratio (SHR_{coil}):

$$SHR_{coil} = \frac{\dot{Q}_{coil,sensible}}{\dot{Q}_{coil}} \quad (16)$$

where $\dot{Q}_{coil,sensible}$ and $\dot{Q}_{coil,latent}$ represent the coil sensible heat and coil latent heat, respectively. They can be calculated as:

$$\dot{Q}_{coil,sensible} = \dot{m}_{a,2}(h'_1 - h_2) \quad (17)$$

$$\dot{Q}_{coil,latent} = \dot{m}_{a,2}(h_1 - h'_1) \quad (18)$$

Figure 2 shows the psychrometric processes for the schematic presented in Figure 1.

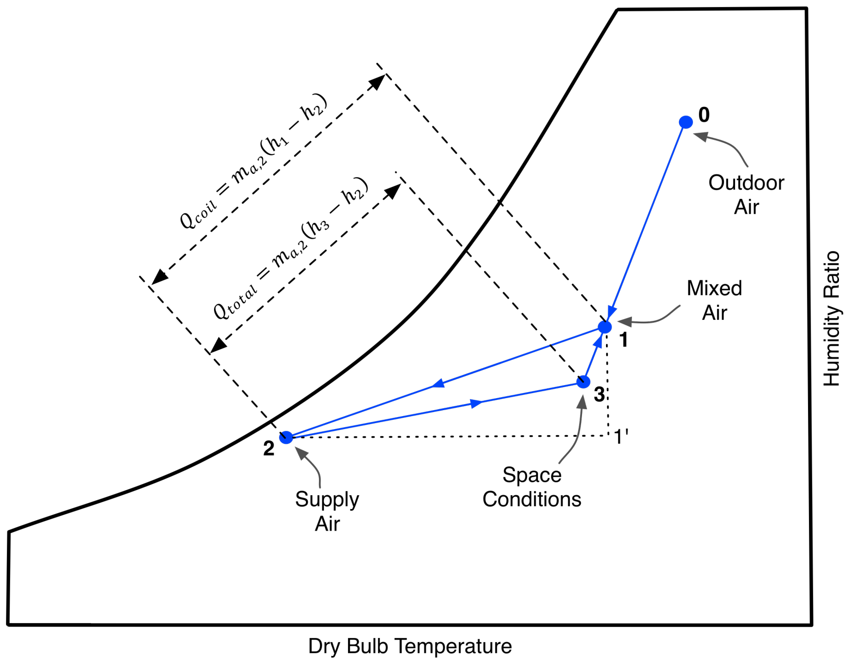

Figure 2. Phychrometric processes for the schematic presented in Figure 1.

TOOLS DEVELOPED IN MATHCAD AND EXCEL

Figure 3 shows a portion of a Mathcad file where students are asked to define basic psychrometric functions that are used in the air conditioning analysis. This first task allows students to gain an understanding of the basic psychrometric processes by creating specific functions in Mathcad/Excel. For instance, students are asked to define the enthalpy as a function of temperature and relative humidity or, alternately, as a function of temperature and humidity ratio. Next students are asked to use the functions created and shown in Figure 3 to analyze the air conditioning problem described in Figure 1. Figure 4 shows the analysis as performed in Mathcad. Finally, Figure 5 presents a screenshot of the tool developed in Mathcad to analyze the air conditioning problem described in Figure 1. This tool is completely based on the Mathcad definitions and derivations described in Figures 3 and 4. After developing this tool, students can vary the input design conditions as well as the load for the space to size the equipment and instantly observe the changes throughout the system. Boxes are drawn around each of the inputs so this information can be easily accessed and modified. The plot shown in Figure 5 illustrates how the modifications can instantly change the process as would be described in a psychrometric chart. In developing this plot, students are guided to think more carefully about the elements of the psychrometric chart: they learn how to draw the saturation line, lines of constant enthalpy, lines of constant humidity ratio, and the path described by the process being examined. In addition, after the initial problem has been solved, students can easily change different variables such as the design conditions and the outdoor conditions to study the effect of these parameters on the equipment selection. Although the above can be done by hand using the psychrometric chart, it is laborious and time consuming and does not provide the student with clear, instantaneous results regarding the effects of changing a specific parameter. It is possible and straight-forward to perform the same analysis in Excel. Figure 6 shows a screenshot of a tool developed in Excel for the same air conditioning problem described in Figure 1. The example below illustrates the use of the developed tool as shown in Figures 5 and 6.

Example

A given space is to be maintained at 25.5°C and 50% relative humidity. The total heat gain to the space has been determined to be 35 kW, of which 24.5 kW is sensible heat transfer (Room sensible heat ratio of 0.7). The outdoor air requirement of the occupants is 1000 L/s. The outdoor air has a temperature and relative humidity on 35°C and 55%, respectively. Determine the quantity and the state of the air supplied to the space and the required capacity of the cooling and dehumidifying equipment.

CONCLUSIONS

The paper has demonstrated how to use Mathcad and/or Excel to teach students to implement a set of psychrometric functions which are then used to reduce the calculation effort from the user to tackle design oriented problems. The objective of this approach has been to enhance the students' appreciation of the basic psychrometric processes and to show them how to create automated worksheets for specific air conditioning applications. Thus, the students are able to immediately determine results due to variations in input design conditions and/or different system parameters. Feedback from students and class instructors indicate that the use of these tools enhances the student experience in the air conditioning class.

1. Saturation Pressure (kPa) as a function of Temperature

$C_1 := -5800.2206$ $\quad\quad$ $C_4 := 4.1764768 \cdot 10^{-5}$

$C_2 := 1.3914993$ $\quad\quad$ $C_5 := -1.4452093 \cdot 10^{-8}$

$C_3 := -0.048640239$ $\quad\quad$ $C_6 := 6.5459673$

$$P_{wsK}(T) := e^{\left(C_1 \cdot \frac{1}{T} + C_2 + C_3 \cdot T + C_4 \cdot T^2 + C_5 \cdot T^3 + C_6 \cdot \ln(T)\right)} \cdot Pa \quad \text{use T in Kelvins}$$

$$P_{wsC}(T) := P_{wsK}(T + 273.15) \quad \text{use T in °C}$$

2. Humidity Ratio

$$w(P_w) := 0.62198 \cdot \frac{P_w}{P_{total} - P_w}$$

$$P_w(w) := \frac{P_{total} \cdot w}{w + 0.62198}$$

$$w_{T\varphi}(T, \varphi) := w(\varphi \cdot P_{wsC}(T)) \quad \text{use T in °C}$$

3. Relative Humidity

$$\varphi = \frac{P_w}{P_{ws}} \qquad Pw_{T\varphi}(T, \varphi) := \varphi \cdot P_{wsC}(T) \qquad \varphi_{Tw}(T, w) := \frac{P_w(w)}{P_{wsC}(T)} \quad \text{use T in °C}$$

4. Specific Volume

$$\nu(T, w) := 287.1 \cdot (T + 273.15) \cdot \frac{(1 + 1.6078 \cdot w)}{\frac{P_{total}}{Pa}} \cdot \frac{m^3}{kg} \quad \text{use T in °C}$$

5. Enthalpy relationships

$$h_{Tw}(T, w) := [1.006 \cdot T + w \cdot (2501 + 1.86 \cdot T)] \cdot \frac{kJ}{kg} \quad \text{use T in °C}$$

$$h_{T\varphi}(T, \varphi) := h_{Tw}(T, w_{T\varphi}(T, \varphi)) \quad \text{use T in °C}$$

$$w_{Th}(T, h) := \frac{h \cdot \frac{kg}{kJ} - 1.006 \cdot T}{2501 + 1.86 \cdot T} \quad \text{use T in °C}$$

6. Sensible and latent heats

$$Q_s(RSHR, Q_{total}) := RSHR \cdot Q_{total}$$

$$Q_L(RSHR, Q_{total}) := Q_{total} - Q_s(RSHR, Q_{total})$$

Figure 3. Definitions of basic psychrometric functions using Mathcad

1. Conditions at State "3" (T_3 and φ_3 known)

$P_{ws3} := P_{wsC}(T_3) = 3.265 \cdot kPa$

$P_{w3} := Pw_{T\varphi}(T_3, \varphi_3) = 1.632 \cdot kPa$

$w_3 := w_{T\varphi}(T_3, \varphi_3) = 0.01022$

$h_3 := h_{T\varphi}(T_3, \varphi_3) = 51.693 \cdot \frac{kJ}{kg}$

2. Conditioned Space: sensible and latent heats

$Q_{sensible} := Q_s(RSHR, Q_{total}) = 24.5 \cdot kW$

$Q_{latent} := Q_L(RSHR, Q_{total}) = 10.5 \cdot kW$

3. Conditions at State "2"

$T_2 := T_3 - \Delta T = 15.5$

Finding h2 using Q_s and RSHR

$$RSHR = \frac{Q_s}{Q_{total}} = \frac{m' \cdot q_s}{m' \cdot q_{total}} = \frac{q_s}{(h_3 - h_2)}$$

$$q_s := 1.005 \cdot (T_3 - T_2) \cdot \frac{kJ}{kg} = 10.05 \cdot \frac{kJ}{kg}$$

$$h_2 := h_3 - \frac{q_s}{RSHR} = 37.336 \cdot \frac{kJ}{kg}$$

Finding w_2, φ_2, and ν_2:

$w_2 := w_{Th}(T_2, h_2) = 0.00859$

$\varphi_2 := \varphi_{Tw}(T_2, w_2) = 0.781666$

Finding the required air flowrate at location 2

$$m'_{a2} := \frac{Q_{total}}{h_3 - h_2} = 2.438 \frac{kg}{s}$$

$$v'_2 := m'_{a2} \cdot \nu(T_2, w_2) = 2.028 \frac{m^3}{s}$$

4. Conditions at State "0"

$w_0 := w_{T\varphi}(T_0, \varphi_0) = 0.01966$

$h_0 := h_{T\varphi}(T_0, \varphi_0) = 85.67 \cdot \frac{kJ}{kg}$

$\nu_0 := \nu(T_0, w_0) = 0.90363 \frac{m^3}{kg}$

Mass flowrate of dry air $\quad m'_{a0} := \frac{V_0}{1000 \cdot \nu_0} = 1.107 \frac{kg}{s}$

5. Recirculated air flowrate at location "4"

$m'_{a4} := m'_{a2} - m'_{a0} = 1.33117 \frac{kg}{s}$

6. Conditions at state "1"

$$h_1 := \frac{m'_{a0} \cdot h_0 + m'_{a4} \cdot h_3}{m'_{a0} + m'_{a4}} = 67117.16 \frac{m^2}{s^2}$$

$$T_1 := \frac{T_1 - T_3}{T_0 - T_3} = \frac{h_1 - h_3}{h_0 - h_3} \;\Bigg|\begin{matrix} \text{explicit} \\ \text{solve}, T_1 \end{matrix} \rightarrow \frac{T_0 \cdot h_1 - T_0 \cdot h_3 + T_3 \cdot h_0 - T_3 \cdot h_1}{h_0 - h_3} = 29.81$$

$w_1 := w_{Th}(T_1, h_1) = 0.014522$

$P_{w1} := P_w(w_1) = 2.304 \cdot kPa$

$\varphi_1 := \varphi_{Tw}(T_1, w_1) = 0.548597$

7. Cooling Coil Calculations

$Q_{coil} := m'_{a2} \cdot (h_1 - h_2) = 72.6 \cdot kW \qquad ton := \frac{kW}{0.284345136}$

$Q_{coil} = 20.6\,ton$

Coil Sensible heat ratio

$h'_1 := h_{Tw}(T_1, w_2) = 51.963 \cdot \frac{kJ}{kg}$

$q_{coil.sensible} := (h'_1 - h_2) \cdot m'_{a2} = 35.7 \cdot kW$

$q_{coil.latent} := (h_1 - h'_1) \cdot m'_{a2} = 36.9 \cdot kW$

$$coil_{SHR} := \frac{q_{coil.sensible}}{q_{coil.sensible} + q_{coil.latent}} = 0.491$$

8. Plotting the results:

Data for lines relating the states

State 2 - State 1 $\quad S21 := \begin{pmatrix} w_2 & T_2 \\ w_1 & T_1 \end{pmatrix}$

State 3 - State 0 $\quad S30 := \begin{pmatrix} w_3 & T_3 \\ w_0 & T_0 \end{pmatrix}$

State 2 - State 3 $\quad S23 := \begin{pmatrix} w_2 & T_2 \\ w_3 & T_3 \end{pmatrix}$

Data for lines of constant enthalpy and constant humidity ratio

$i := 0..50 \qquad Temp_i := 10 + 50 \cdot \frac{i}{50}$

$$h_{constant}^{\langle i \rangle} := \left(w_{Th}(Temp_i, h_3) \;\; w_{Th}(Temp_i, h_2) \;\; w_{Th}(Temp_i, h_1) \;\; w_{Th}(Temp_i, h_0) \right)^T$$

$T_h := augment(Temp, Temp, Temp, Temp) \qquad h_{constant} := h_{constant}^T$

$$w_{constant}^{\langle i \rangle} := \left(w_{T\varphi}(Temp_i, \varphi_3) \;\; w_{T\varphi}(Temp_i, \varphi_0) \;\; w_{T\varphi}(Temp_i, \varphi_2) \right)^T$$

$T_w := augment(Temp, Temp, Temp) \qquad w_{constant} := w_{constant}^T$

Figure 4. Air Conditioning Analysis using Mathcad

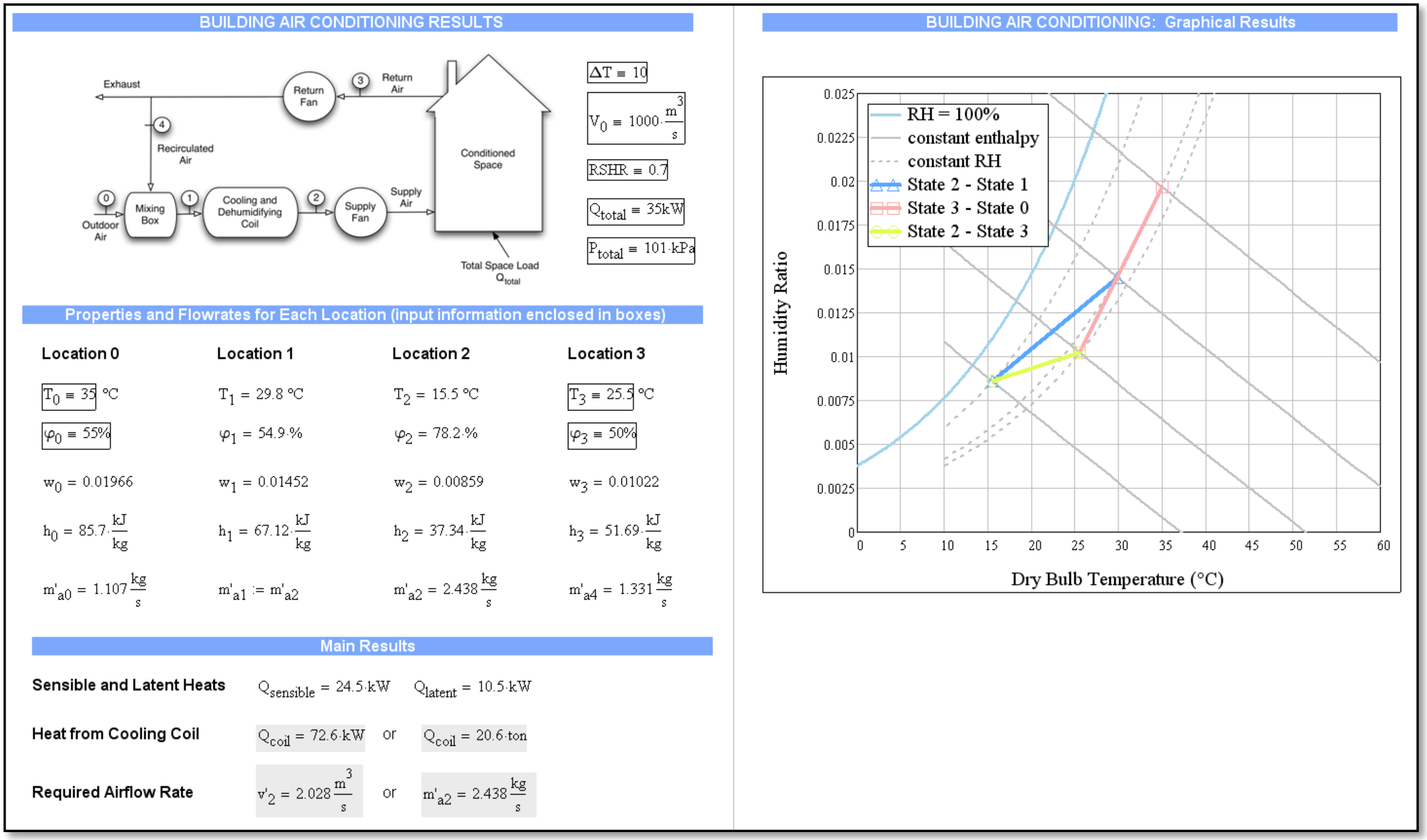

Figure 5. Screenshot for the tool developed in Mathcad

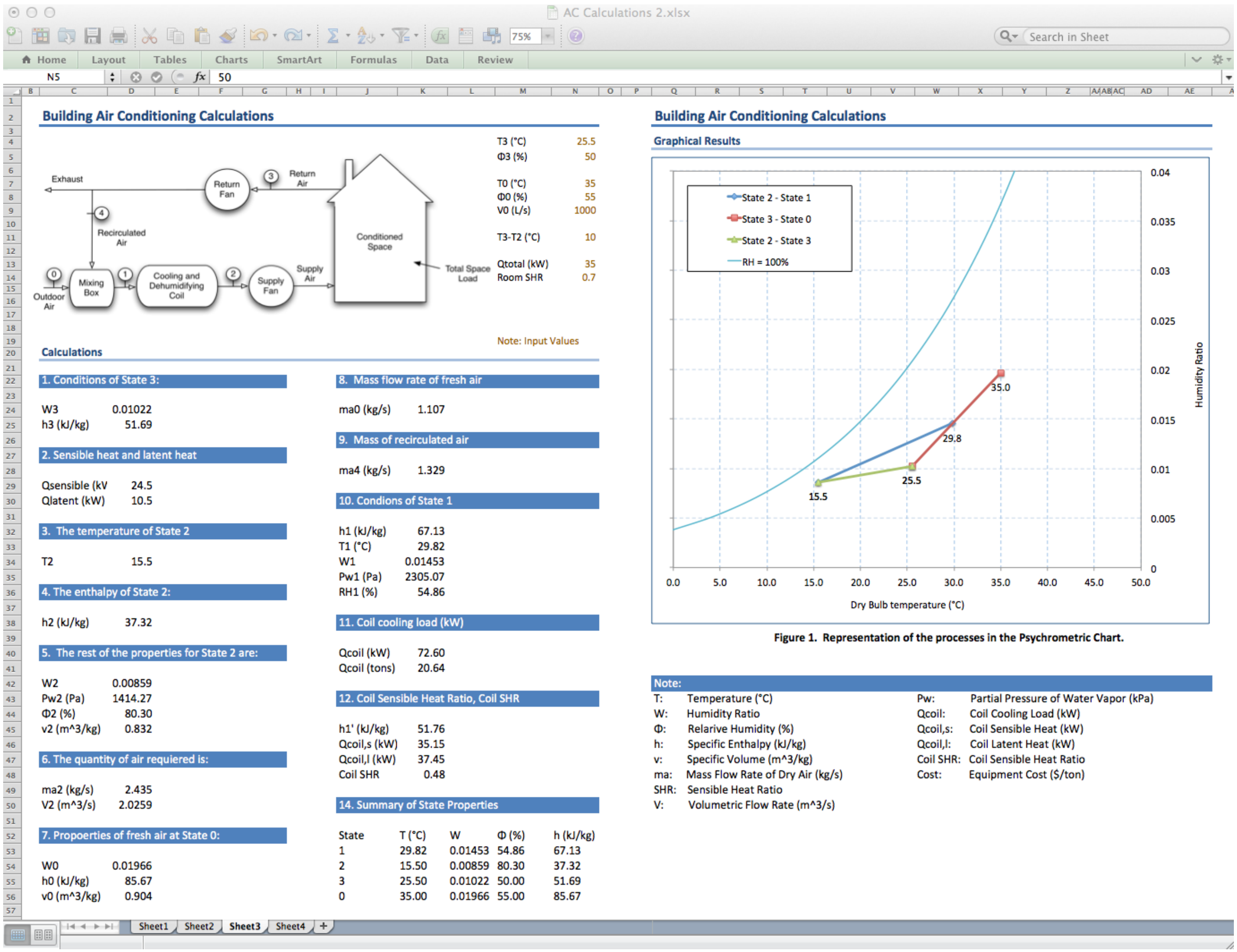

Figure 6. Screenshot for the tool developed in Excel

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