

Application of Lyapunov Exponents to Damaged Ship Stability Cases

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Introduction

Ship capsizing is a chaotic phenomenon; this may be the most apt description for a process that cannot yet truly be predicted in random sea states because of the sensitivity to initial conditions. Many mathematical and statistical methods have been employed to determine the probabilities of ship capsizing in certain sea conditions, including programs such as the U.S. Navy's SMP, CRNAV and Marin's FREDYN, and SAIC's LAMP. However, because of the chaotic nature of the problem there is no way, as of yet, to completely accurately predict ship capsizing in a series of random waves. Recent innovation in hull design has been heightening the awareness of these issues. This very real problem is where the mathematical study of chaotic processes may one-day allow for the real-time prediction of whether a ship is facing imminent capsizing.

Lyapunov Exponents

The author uses a mathematical technique called the Lyapunov exponent to study the motion of ships in extreme sea conditions. The Lyapunov exponent is basically a measure of the "stretching" of a group of points in a phase space based on their initial conditions. This stretching can come in the form an expanding or contracting nature, and may best be visualized by a ball of initial condition points. Because of the local deformations, or stretching, of the flow, the ball of initial condition points in k dimensions will become an k -dimensional ellipsoid whose axes are deforming exponentially as defined by these Lyapunov exponents. Therefore, the exponent provides a measure for the separation of initial conditions points as they evolve in time. This technique can be applied to many different types of systems, including the complex nonlinear dynamics of a ship facing capsizing in random seas.

Application to Damaged Ship Stability

For this study the author used the Finite Time Lyapunov Exponent (FTLE), which simply defines the exponent over a short time interval instead of an infinite time series, to study the motions of a ship in a damaged ship stability case. A MATLAB code was used that calculated the Lyapunov exponent and FTLE for Roll and Pitch vs. Time for numerous runs of approximately 2,500 seconds long. It was found that there was strong correlation between large amplitude ship motions and spikes in the numerical value of the FTLE. This promising data shows that the Lyapunov exponents may very well be a valid way to predict the advent of large amplitude ship motions in a chaotic system such as a damaged stability case.

Conclusion

The nonlinear motions of a ship in a random seaway is a complex problem that is difficult to solve. This method shows potential for being able to predict the advent of large amplitude ship motions that could lead to catastrophic results, such as ship capsizing. The Lyapunov exponent certainly holds promise to being able to predict these motions, and this research will hopefully help validate its use in the field of nonlinear ship dynamics.