Using Design-Expert for Enhancing Engineering Experimentation Labs

Y. Charles Lu¹, William E. Murphy¹, Vincent R. Capece¹

Abstract - At the University of Kentucky, College of Engineering Extended Campus in Paducah, all mechanical engineering students are required to take a series of experimentation courses: Engineering Experimentation I (ME310) and Engineering Experimentation II (ME311). The Engineering Experimentation II is an advanced course dealing with principles of experimental design and analysis known as DOE. The DOE involves heavily combinatorial design and statistical analysis. The Design-Expert software has been integrated into this laboratory course and enhanced effectively the students’ abilities to design and analyze experiments.

Keywords: Design-Expert, Design of experiments, Statistical analysis, Laboratory

Introduction

At the University of Kentucky, College of Engineering Extended Campus in Paducah, all mechanical engineering students are required to take a series of experimentation courses: ME310-Engineering Experimentation I and ME311-Engineering Experimentation II. While ME310 focuses on fundamentals of measurement techniques, instrumentation, interfaces, etc., ME311 focuses more on the overall experimentation process including experimental design, execution and analysis known as design of experiment (DOE).

The design of experiments relies on principles of combinatorial mathematics such as combination, permutation, factorial, blocking and Latin square [1, 2, 3]. It is often hard for the students to manually design an experimental layout if they don’t have sufficient combinatorial background.

The analysis of experiments uses theories from statistics such as hypothesis, t-test, F-test and ANOVA [1, 2, 3]. The theories of statistical analysis are generally easy for students to grasp, but the calculations are often tedious and consume a significant amount of class time. That could potentially shift the course focus away from experimentation and jeopardize the students’ interests in these courses.

We have recently integrated the Design-Expert software into the ME311 course. This software is developed by Design-Ease, Inc. for experimental design and analysis [4]. It comes with the textbook Design and Analysis of Experiments, by D. C. Montgomery, published by Wiley [1]. The Design-Expert software is a Windows-based program with many powerful DOE tools, such as two-level factorial screening, general factorial design, response surface methods, mixture design and Taguchi design [4]. With the use of this software, the lecture contents have been presented more effectively. Students have used this software for designing and analyzing their experiments. As a result, the overall quality of the lab experiments has been significantly improved.

¹ College of Engineering Extended Campus, University of Kentucky, 4810 Alben Barkley Drive, Paducah, KY 42002.
Example Lecture with Design-Expert

An example lecture on a one-factor factorial experiment, the hardness experiment, is used to demonstrate a very important design technique called “Blocking”. In any experiment, variability arising from a nuisance factor can affect the results. If the nuisance factor is known and controllable, then “Blocking” can be used to systemically eliminate its effect on variations among experiments.

*A hardness machine is used to measure the hardness of a material. There are four indenter tips available and we wish to determine whether or not the four different indenter tips produce different hardness readings. Since the surface quality of test coupon can greatly affect the experimental results, the test coupon has to be considered as a block.*

**Design of experiments**
The experiments can be designed based on the following parameters:

- Factor: indenter tip
- Level: 4 different tips
- Block: specimen
- Replication: 4
- Randomization: Yes

Since there are four (4) tips and each tip needs to be tested four (4) times, there are a total of $4 \times 4 = 16$ tests. Within each block, there are $4! = 24$ possible arrangements. For the four (4) blocks, the total number of ways of arranging the experiments are: $24 \times 4 = 96$. One of the experimental layouts is shown in Figure 1.

![Figure 1. Design Layout for Hardness Experiments involving Blocking and Randomization.](image)

The above experimental design can be better illustrated with Design-Expert. First, we select General Factorial and then choose Categorical Factors as 1, as indicated in Figure 2.

![Figure 2. Design of Hardness Experiments with Design-Expert: Defining Factors](image)
Then, we can define the Factor Name and Unit, and then the Factor Levels, as seen in Figure 3.

![Figure 3. Design of Hardness Experiments with Design-Expert: Defining Levels](image)

Since we want to have replications, we can select the numbers of replicates we desire. We can also check whether or not to assign “blocking” to the experiment, as seen in Figure 4.

It is noted that students are often confused by the concept of “Blocking”. Blocking is an extremely important and useful “mathematical” or “statistical” technique in designing an experiment. It is intended for minimizing or eliminating the effect of “nuisance factors” or “noise factors” that occur in the experiments, such as variations from raw materials, specimens, operators and environment. Different statistical analyses will be used for experiments with and without blocking. Thus, it is very critical here to consider whether or not to check the “blocking” box.

![Figure 4. Design of Hardness Experiments with Design-Expert: Defining Replications and Blocking.](image)

Once all parameters/factors/responses are considered, the software will generate a “standard” design layout. If we want to further eliminate additional errors that may come from other sources such as the equipment, we can randomize the test sequence by conducting “randomize by run order”. As discussed earlier, the total number of ways of arranging the experiments is 24x4= 96, so it is highly likely that each student will obtain a different design layout. Figure 5 shows one of the experimental layouts after randomization.
Execution of experiments

Once the experimental design is completed, we can implement the design layout in the lab and then conduct the tests and collect the data. For this example problem, the hardness data from one of the experimental layouts are given in Table 1.

Table 1. Hardness Data Obtained from Four (4) Tips.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Tip 1</th>
<th>Tip 2</th>
<th>Tip 3</th>
<th>Tip 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.3</td>
<td>9.4</td>
<td>9.6</td>
<td>10.0</td>
</tr>
<tr>
<td>2</td>
<td>9.4</td>
<td>9.3</td>
<td>9.8</td>
<td>9.9</td>
</tr>
<tr>
<td>3</td>
<td>9.2</td>
<td>9.4</td>
<td>9.5</td>
<td>9.7</td>
</tr>
<tr>
<td>4</td>
<td>9.7</td>
<td>9.6</td>
<td>10.0</td>
<td>10.2</td>
</tr>
</tbody>
</table>

Analysis of experiments

The most challenging task in the experimentation process is probably data analysis. For factorial type experiments, the ANOVA (analysis of variance) is typically used. The general procedure for conducting an ANOVA consists of three steps:

1. Set up hypotheses:
   - Null hypothesis: \( H_0 : \mu_1 = \mu_2 = \ldots = \mu_a \)
   - Alternative hypothesis: \( H_1 : \mu_i \neq \mu_j \)

2. Perform statistic test:
   \[
   F_0 = \frac{\text{MS}_{\text{Treatment}}}{\text{MSE}}
   \]

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(3) Draw conclusions:

- compare $F_0$ to $F_{a-1,(a-1)(b-1)}$, the critical value in F-distribution
- if $|F_0| > F_{a-1,(a-1)(b-1)}$, the null hypothesis is rejected and the alternative hypothesis is accepted.

For this experiment, we first set up the hypothesis as follows:

Null hypothesis: all tips produce the same hardness number
Alternative hypothesis: one tip produces different hardness number from the other tips

The statistical test is performed in the ANOVA analysis, as summarized in Table 2.

### Table 2. The ANOVA Table for One-factor Experiment

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>$F_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatments</td>
<td>$SS_{T}$</td>
<td>$a-1$</td>
<td>$SS_{T}/(a-1)$</td>
<td>$MS_{T}/MS_E$</td>
</tr>
<tr>
<td>Blocks</td>
<td>$SS_{blocks}$</td>
<td>$b-1$</td>
<td>$SS_{blocks}/b-1$</td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>$SS_E$</td>
<td>$(a-1)(b-1)$</td>
<td>$SS_E/(a-1)(b-1)$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$SS_T$</td>
<td>$N-1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The variables in Table 2 are defined by the following equations.

- $y_i = \sum_{j=1}^{a} \sum_{j=1}^{n} y_{ij}$
- $\bar{y}_i = y_i / N$
- $SS_T = \sum_{i=1}^{a} \sum_{j=1}^{n} y_{ij}^2 - \frac{y_i^2}{N}$
- $SS_{blocks} = \frac{1}{a} \sum_{i=1}^{b} \sum_{j=1}^{n} y_{ij}^2 - \frac{y_i^2}{N}$
- $SS_{T} = \frac{1}{n} \sum_{i=1}^{a} \sum_{j=1}^{n} y_{ij}^2 - \frac{y_i^2}{N}$
- $SS_E = SS_T - SS_{T}$

Once the analysis is completed, we can draw a conclusion on if the hypothesis is accepted or rejected. If rejected, it means that one tip performs differently from the others. Next, we would like to know if any pair of tips performs the same or differently. That requires that the Fisher’s LSD (Least Significant Difference) test be performed, which is essentially the multiple paired t-tests.

$$LSD = \frac{1}{\sqrt{\frac{2MS_E}{n}}}$$

where $t_{a/2, (a-1)(b-1)}$ is the critical value from t-distribution and $MS_E$ is the mean square of error available in Table 2.

If $|\bar{y}_i - \bar{y}_j| > LSD$, we conclude that the means $\mu_i$ and $\mu_j$ differ.

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Although the principles of statistical analysis have been covered to some extent in the first experimentation course (ME310) and are thus relatively easy for students to understand, the actual calculations do consume a lot of class time and could make lectures boring.

With Design-Expert, the ANOVA can be performed very easily. First, we need to enter the experimental results to the Response column in the design table (Figure 6). Then, all analyses, diagnostics, plots are generated immediately.

![Table 3. The ANOVA Table for One-Factor Hardness Experiments from Design-Expert.](image)

Table 3 shows an example of the ANOVA table from Design-Expert, which is in the same form as Table 2. The conclusion on the hypothesis test is also given as indicated by “significant” or “insignificant”.

Figure 6. The Hardness Numbers from Four (4) Tips.
Table 4 below summarizes the Fisher’s LSD calculations. It is basically six paired t-tests. Based on this, we can conclude that tips 1 and 2, 1 and 3, 2 and 3 are essentially the same, while tips 1 and 4, 2 and 4, 3 and 4 are different. The effect of indenter tip on hardness can also be graphed as seen in Figure 7. The validity of the analysis can be verified by plotting out the normal probability of the residuals (Figure 8).

| Treatment | Difference | DF | Error | Coeff-θ | Prob > | | Standard | t for H₀ |
|-----------|------------|----|-------|---------|--------|
| 1 vs 2    | -0.025     | 1  | 0.067 | -0.30   | 0.7163 |
| 1 vs 3    | 0.13       | 1  | 0.067 | 1.67    | 0.0935 |
| 1 vs 4    | -0.30      | 1  | 0.067 | -4.50   | 0.0015 |
| 2 vs 3    | 0.15       | 1  | 0.067 | 2.25    | 0.0510 |
| 2 vs 4    | -0.27      | 1  | 0.067 | -4.12   | 0.0026 |
| 3 vs 4    | -0.43      | 1  | 0.067 | -6.37   | 0.0001 |

Figure 7. The Hardness Number vs. Indenter Tip.

Figure 8. The Normality Plot of Hardness Experiments.

Example Lab with Design-Expert

Design-Expert software has been used not just as an instructional tool, but also for performing the actual experiments. In this course, it is consistently stressed that the complete experimentation process consists of three steps: design, execution and analysis. So, students are encouraged to use Design-Expert for all their experiments.
Here is an example experiment performed by students. It is a $2^2$ factorial experiment involving vibration response of a steel plate. In this experiment, a 1/8” thick steel plate was used for resonant frequency measurements. The experiments included striking the plate with an impact hammer and then measuring the response with accelerometers. The first three modal frequencies of a smooth plate were determined as 698 Hz, 1216 Hz and 2125 Hz. Then the students were asked to optimize the resonant responses of the plate by adding structural ribs. Students had used Design-Expert to investigate two factors: (A) rib size and (B) rib location. The experimental design is seen in Table 5. Figure 9 illustrates some of the rib patterns designed by students.

Table 5. Design Layout and Results for the $2^2$ Factorial Vibration Experiments.

<table>
<thead>
<tr>
<th>Run</th>
<th>Factor 1 A. Rib Size</th>
<th>Factor 2 B. Rib Position</th>
<th>Response 1 1st Modal Freq. Hz</th>
<th>Response 2 2nd Modal Freq. Hz</th>
<th>Response 3 3rd Modal Freq. Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Big</td>
<td>Parallel</td>
<td>781.0</td>
<td>1422.0</td>
<td>1747.0</td>
</tr>
<tr>
<td>2</td>
<td>Small</td>
<td>Parallel</td>
<td>722.0</td>
<td>1288.0</td>
<td>1981.0</td>
</tr>
<tr>
<td>3</td>
<td>Big</td>
<td>Cross</td>
<td>865.0</td>
<td>1281.0</td>
<td>2431.0</td>
</tr>
<tr>
<td>4</td>
<td>Small</td>
<td>Cross</td>
<td>772.0</td>
<td>1194.0</td>
<td>2191.0</td>
</tr>
</tbody>
</table>

After measuring the modal frequencies of those modified plates, the students entered the experimental data back into the software as shown in Table 5. Then the students went to the “Analysis” step and obtained the statistical analysis results. The ANOVA results for the 1st modal frequency are summarized in Table 6. It shows that both factors, (A) rib size and (B) rib position, have significant effect on the first mode of the plate. Figure 10 shows the same effect graphically.

Table 6 – ANOVA Results for the 1st Modal Frequency

<table>
<thead>
<tr>
<th>Response: 1st Modal Frequency</th>
<th>ANOVA for Selected Factorial Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source</td>
<td>Sum of Squares</td>
</tr>
<tr>
<td>A</td>
<td>5.8E+003</td>
</tr>
<tr>
<td>B</td>
<td>4.5E+003</td>
</tr>
</tbody>
</table>

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Conclusions

Design-Expert software has been integrated into an advanced engineering experimentation course that deals with design, execution and analysis of experiments known as DOE. Design-Expert has been found to be an effective tool for illustrating the DOE principles and performing the statistical analysis. The amount of time spent on “abstract” combinatorial design and statistical analysis can be greatly minimized. Students have used this software for designing and analyzing their experiments. The overall quality of their experiments has been greatly improved.

References


Y. CHARLES LU
Y. Charles Lu is an Assistant Professor of Mechanical Engineering. Dr. Lu received his Ph.D. and M.E. Sc in Mechanical Engineering from the University of Western Ontario, Canada, M. Eng in Materials Engineering from NEFU, China and M.S. in Mathematics from Murray State University.

VINCENT R. CAPECE
Vincent R. Capece is an Associate Professor of Mechanical Engineering. Dr. Capece received his B.S. degree in Mechanical Engineering from Tennessee Technological University in 1980, M.S. in Mechanical Engineering from MIT in 1982, and Ph.D. from Purdue University in 1987.

WILLIAM E. MURPHY
William E. Murphy is Director of the Paducah Engineering Extended Campus and Professor of Mechanical Engineering with the University of Kentucky. He received his BS degree from the University of Kentucky and his MS and PhD degrees from Purdue, all in Mechanical Engineering. He has been active in engineering accreditation issues for 15 years.