Effectiveness and LMTD Correction Factor of the Cross Flow Exchanger:
a Simplified and Unified Treatment

Sheldon M. Jeter [1]

Abstract – A unified and somewhat simplified although complete presentation of the performance of the most common cross flow heat exchanger is presented. The results from effectiveness and LMTD analysis are presented in enough detail that a real appreciation for the analysis and an understanding of the application of the results can be developed. The presentation also includes the development of the formula for the mean temperature difference, a concept or methodology no longer much in use. The usual plots of the effectiveness, geometrical correction factor, and mean temperature difference are presented; however, in this case interested professors and students can obtain the spreadsheet used to generate the data from the author.

Keywords: Heat Exchangers, Cross Flow

INTRODUCTION

The cross flow exchanger is probably the dominant heat exchanger type in overall usage. For examples, cross flow exchangers are ubiquitous in heating, ventilating, and air conditioning (HVAC) systems not only as cooling and dehumidification coils but also as heating coils and air-cooled condensers. They are also commonly encountered as vehicle engine radiators and in other process and component cooling and heating applications. In contrast, shell and tube exchangers are relatively uncommon, being mostly restricted to purely industrial applications. The cross flow exchanger is thus an important everyday device, while the shell and tube exchanger is less familiar and somewhat exotic to the typical undergraduate. For its practical importance and for the inherent educational benefit, the cross flow exchanger deserves enhanced treatment in undergraduate courses.

Despite its practical importance, the cross flow exchanger is usually de-emphasized in most textbooks and presumably in classroom presentations. Indeed most instructors probably only cover the simple counter flow and parallel flow exchangers. Ideally, if not in a required heat transfer course at least in an elective thermal systems course, one would also discuss the U-tube exchanger and at least one cross flow exchanger. The most representative cross flow exchanger has only one fluid mixed. Treatment of this group along with general discussion of the more complicated alternative designs would provide an adequate introduction to heat exchanger theory for today's undergraduate.

This paper presents a somewhat simplified and unified derivation of the effectiveness and the geometrical correction factor for the cross flow exchanger with one fluid mixed. The present author has found this treatment worthwhile for several reasons. One major reason is that the application of existing and well established formulas is really not always entirely straightforward unless one has a firm understanding of the underlying technology, theory, and analysis. Indeed, one can find textbooks in which the well known formulas and charts are misapplied. A minor reason is that an error was found in an historically important paper that probably should be corrected.

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[1] Associate Professor, The George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0405, sheldon.jeter@me.gatech.edu.

The present development proceeds first to derive the conventional effectiveness of this particular cross flow exchanger. This derivation includes reference to the important well known behavior for heat transfer when one temperature is fixed. This reference is felt to be an important unifying concept. The next approach is to find the mean temperature for heat transfer. This method is historically significant and is quite simple. Students may find this approach to be a reasonable alternative when confronting unusually complex situations in their own work. The final development is to derive a formula for the geometrical correction factor from the effectiveness. This derivation unifies several heat exchanger concepts and results in a simple expression that can be compared and contrasted with the results for counter flow and parallel flow that are already usually presented. The educational benefits from including three distinct but related approaches in such a presentation are reinforcement and unification, and students will also benefit from extra emphasis on an exchanger of great practical significance.

As seen below, analysis of the cross flow exchanger is straightforward but worthy of classroom time. In the next section, the two popular methods of heat exchanger analysis, the effectiveness and the LMTD methods, will be presented along with the historically significant mean temperature difference method.

ANALYSIS BY THE EFFECTIVENESS METHOD

The first presentation is the conventional heat exchange effectiveness method. This analysis is very well known and is universally adopted as one of the preferred methods of presenting and conducting heat exchanger analysis. The general effectiveness method appears to be first publicized by London and Seban (1942). Specific results for cross flow exchangers appear in the classic text by Kays and London (1964) if not well before. The presentation and derivation in this paper is not asserted to be very original, but it is thorough and complete enough for undergraduates to follow on their own or to support an easy classroom presentation. In this presentation, reference is made to some unifying themes common to exchanger analysis.

The pertinent geometry for this analysis is shown in Figure 1. The figure represents what is probably the most common cross flow arrangement, at least in HVAC applications. In this case, the mixed fluid is visualized as the fluid such as a refrigerant, secondary coolant, or hot water flowing inside a heat exchanger tube or pipe. This fluid, flowing inside a small diameter tube or pipe, is sure to be rather well mixed. The unmixed fluid is the fluid, commonly air, flowing across the tubing. Frequently when the outside unmixed fluid is ambient air, it is flowing between fins, which emphasizes its unmixed nature.

In this accompanying figure on the next page, the mixed fluid is shown as flowing in the vertical y direction inside a pipe, and the unmixed fluid is flowing in the horizontal x direction between fins.

In the illustration, heat is assumed to be transferred from the warmer mixed fluid to the unmixed fluid. For thermodynamic analysis, the exchanger is considered to be divided into infinitesimal slices in the y direction. For some differential step in the x direction, the heat transfer rate from some infinitesimal area is

$$\delta^2 \dot{Q} = U\, dA\left(T_{\mathrm{MX}} - T_{\mathrm{UN}}\right) = U\left(dx\, dy\right)\left(T_{\mathrm{MX}} - T_{\mathrm{UN}}\right) \tag{1}$$

Here $\dot{Q}$ is the heat transfer rate, U is the unit conductance, A is the transfer area, T_{MX} is the mixed fluid temperature, T_{UN} is the unmixed fluid temperature, and x and y are linear coordinates. The heat transfer rate is written as a second order differential because it is the heat transfer across the infinitesimal patch of area, $dx\, dy$, which as illustrated in Figure 1 is inherently second order. In any infinitesimal cross section slice, only an infinitesimal fraction of the unmixed fluid flows along the slice whereas the entire finite flow of the mixed fluid flows perpendicular to the slice; consequently, the temperature of the mixed fluid is only a function of y, the flow direction for this fluid. An energy balance on any infinitesimal slice of the unmixed fluid in cross flow gives the following differential formulation.

$$\delta^2 \dot{Q} = \left(\dot{m}_{\mathrm{UN}} \frac{dy}{L_y}\right) C_{\mathrm{UN}}\, dT_{\mathrm{UN}} = U\left(dx\, dy\right)\left(T_{\mathrm{MX}}(y) - T_{\mathrm{UN}}\right) \tag{2}$$

Here $\dot{m}_{\mathrm{UN}}$ is the mass flow rate of unmixed fluid, L_y is the flow length in the y direction, and C_{UN} is the heat capacity of the unmixed fluid. Note that the inlet temperature of the unmixed fluid is presumed to be constant for all values of y. Simplifying gives the following differential equation for the change in the temperature of the unmixed fluid in the x direction.

$$\frac{\partial T_{\mathrm{UN}}}{\partial x} = \frac{U\, L_y}{\dot{m}_{\mathrm{UN}}\, C_{\mathrm{UN}}} \left(T_{\mathrm{MX}}(y) - T_{\mathrm{UN}}\right) \tag{3}$$

This partial differential equation is readily simplified and then integrated as show below.

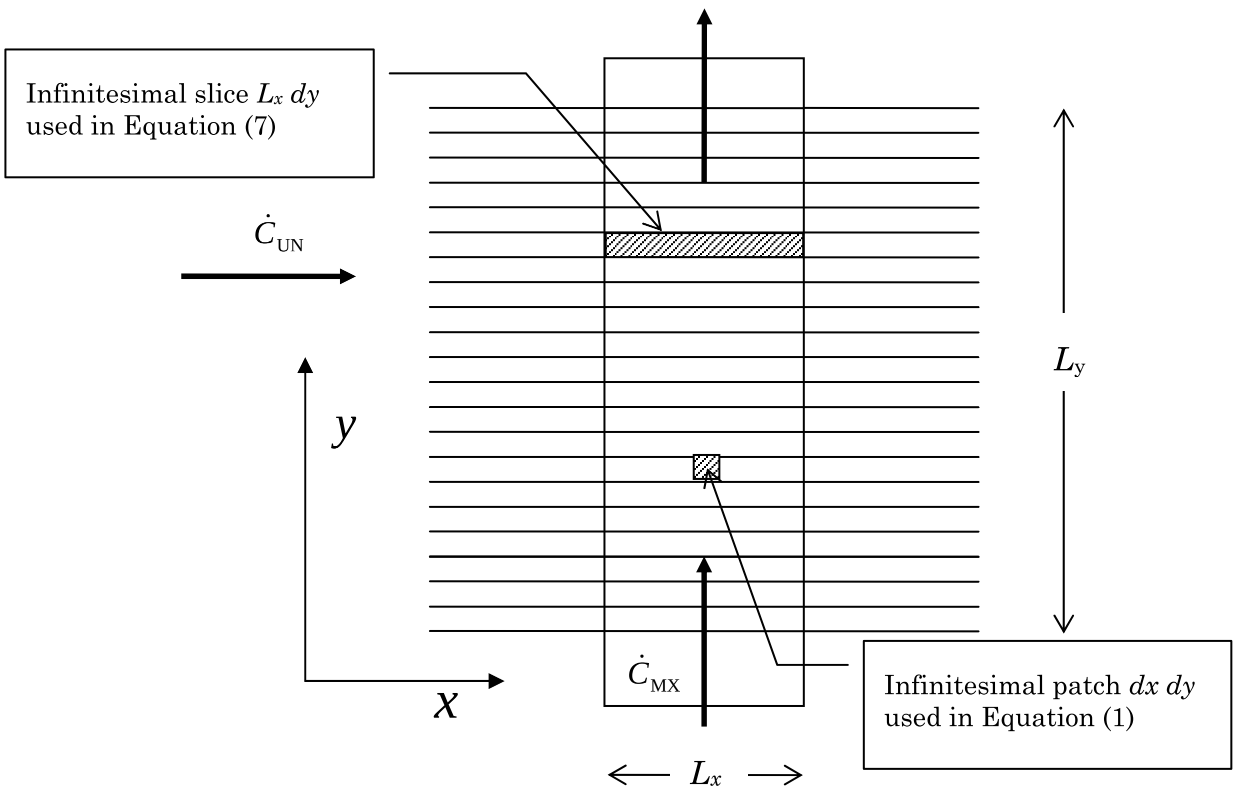

Figure 1. Schematic of Cross Flow Exchanger with Mixed Fluid Shown as Flowing Inside a Single Pipe and the Unmixed Fluid Flowing Outside between Fins

The next step is separating variables and introducing the heat capacity rate, $\dot{C}_{\mathrm{UN}} = \dot{m}_{\mathrm{UN}} C_{\mathrm{UN}}$, for the unmixed fluid giving the following reformulation, which can be reconsidered as an ordinary differential equation at any fixed value of y,

$$\frac{dT_{\mathrm{UN}}}{\left(T_{\mathrm{MX}}(y) - T_{\mathrm{UN}}\right)} = \frac{U\, L_y}{\dot{m}_{\mathrm{UN}}\, C_{\mathrm{UN}}}\, dx = \frac{U\, L_y}{\dot{C}_{\mathrm{UN}}}\, dx \tag{4}$$

This equation can be easily integrated and evaluated to give

$$\ln\left(\frac{\left(T_{\mathrm{MX}}(y)-T_{\mathrm{UN,out}}(y)\right)}{\left(T_{\mathrm{MX}}(y)-T_{\mathrm{UN,in}}\right)}\right)=-\frac{U\,L_y}{\dot{m}_{\mathrm{UN}}\,C_{\mathrm{UN}}}L_x=-\frac{U\,L_y\,L_x}{\dot{C}_{\mathrm{UN}}}=-\frac{U\,A}{\dot{C}_{\mathrm{UN}}} \tag{5}$$

After suitable rearrangement one has

$$\frac{\left(T_{\mathrm{UN,out}}(y)-T_{\mathrm{UN,in}}\right)}{\left(T_{\mathrm{MX}}(y)-T_{\mathrm{UN,in}}\right)}=1-\exp\left(-\frac{U\,A}{\dot{C}_{\mathrm{UN}}}\right) \tag{6}$$

It is now recognized that the preceding equation is just the formula for the effectiveness of a heat exchanger when the temperature of one of the fluids in unchanged. Alternatively, one could just recognize this situation at the start and eliminate the foregoing preliminary analysis. Either way, identify this effectiveness as the local heat exchange effectiveness, ε_{L}, and also write the heat transferred to the mixed fluid in an infinitesimal slice in the *y* direction, as illustrated in Figure 1, as follows

$$\delta\dot{Q}=d\dot{C}_{\mathrm{UN}}\,\left(T_{\mathrm{UN,out}}-T_{\mathrm{UN,in}}\right)=\left(\frac{\dot{C}_{\mathrm{UN}}}{L_y}dy\right)\varepsilon_{\mathrm{L}}\left(T_{\mathrm{MX}}(y)-T_{\mathrm{UN,in}}\right)=-\dot{C}_{\mathrm{MX}}dT_{\mathrm{MX}} \tag{7}$$

As noted above, the local effectiveness ε_{L} is the constant value given by

$$\varepsilon_{\mathrm{L}}=1-\exp\left(-\frac{U\,A}{\dot{C}_{\mathrm{UN}}}\right)=1-\exp\left(-NTU_{\mathrm{UN}}\right) \tag{8}$$

Here the ad hoc number of transfer units NTU_{UN} is especially defined on the basis of the unmixed fluid. No temperature variation with respect to *x* remains in Equation (7), so separating variables gives the following differential equation for the mixed fluid temperature with respect to *y*,

$$\frac{dT_{\mathrm{MX}}}{\left(T_{\mathrm{MX}}(y)-T_{\mathrm{UN,in}}\right)}=-\frac{\dot{C}_{\mathrm{UN}}}{\dot{C}_{\mathrm{MX}}\,L_y}dy\,\varepsilon_{\mathrm{L}} \tag{9}$$

This equation can be easily integrated and evaluated and rearranged to give the overall temperature change for the mixed fluid. Note that the mixed fluid was presumed to be the warm fluid. The result is

$$\frac{\left(T_{\mathrm{MX,in}}-T_{\mathrm{MX,out}}\right)}{\left(T_{\mathrm{MX,in}}-T_{\mathrm{UN,in}}\right)}=1-\exp\left(-\frac{\dot{C}_{\mathrm{UN}}}{\dot{C}_{\mathrm{MX}}}\varepsilon_{\mathrm{L}}\right) \tag{10}$$

The ratio on the right in Equation (10) is common enough and important enough to be give a distinctive ad hoc definition and symbol. For consistence with the literature, use P_{MX} for this ratio. The temperature ratio P_{MX} can be recognized as the heat exchange effectiveness based, not on the minimum fluid, but on the mixed fluid. It is convenient to base such an ad hoc effectiveness in cross flow on the mixed fluid because only a mixed fluid has a well defined single valued outlet temperature. Any effectiveness formula will then be especially simple if written in terms of a mixed fluid, as in this case where

$$P_{\mathrm{MX}}=\frac{\dot{C}_{\mathrm{MX}}\Delta T_{\mathrm{MX}}}{\dot{C}_{\mathrm{MX}}\Delta T_{\mathrm{MAX}}}=\frac{\Delta T_{\mathrm{MX}}}{\Delta T_{\mathrm{MAX}}}=\frac{\left(T_{\mathrm{MX,in}}-T_{\mathrm{MX,out}}\right)}{\left(T_{\mathrm{MX,in}}-T_{\mathrm{UN,in}}\right)}=1-\exp\left(-\frac{\dot{C}_{\mathrm{UN}}}{\dot{C}_{\mathrm{MX}}}\varepsilon_{\mathrm{L}}\right) \tag{11}$$

Note that the temperature ratio P_{MX} is merely the ratio of the change in temperature of the mixed fluid to the maximum temperature difference, which is the difference between inlet temperatures. For future reference, the corresponding temperature ratio of the unmixed fluid is introduced next as

$$P_{\mathrm{UN}} = \frac{\left(T_{\mathrm{UN,out}} - T_{\mathrm{UN,in}}\right)}{\left(T_{\mathrm{MX,in}} - T_{\mathrm{UN,in}}\right)} = \frac{\Delta T_{\mathrm{UN}}}{\Delta T_{\mathrm{MAX}}} \tag{12}$$

To complete the presentation the ad hoc P_{MX} formulation should be related to the conventional effectiveness, which is based on the minimum fluid. When the mixed fluid actually is the minimum fluid, then P_{MX} is the conventional effectiveness. Nevertheless, one notes in this case that the specialized NTU_{UN} that must be used to calculate the local effectiveness is not the conventional NTU or NTU_{MIN} for emphasis, which is of course always defined with respect to the minimum fluid. A heat capacity rate ratio is always needed in effectiveness analysis, so the appropriate heat capacity rate ratio is defined here as

$$\phi_{\mathrm{MX}} = \frac{\dot{C}_{\mathrm{MX}}}{\dot{C}_{\mathrm{UN}}} \tag{13}$$

When the mixed fluid is the minimum fluid and $\phi_{\mathrm{MX}} < 1.0$, the ratio P_{MX} is the conventional effectiveness, so

$$\varepsilon_{\mathrm{XF}} = P_{\mathrm{MX}} = \frac{\dot{C}_{\mathrm{MIN}} \Delta T_{\mathrm{MIN}}}{\dot{C}_{\mathrm{MIN}} \Delta T_{\mathrm{MAX}}} = \frac{\dot{C}_{\mathrm{MX}} \Delta T_{\mathrm{MX}}}{\dot{C}_{\mathrm{MX}} \Delta T_{\mathrm{MAX}}} = 1 - \exp\left(-\frac{\varepsilon_{\mathrm{L}}}{\phi_{\mathrm{MX}}}\right) \tag{14}$$

or after introducing the formula for the local effectiveness

$$\varepsilon_{\mathrm{XF}} = P_{\mathrm{MX}} = 1 - \exp\left(-\frac{1}{\phi_{\mathrm{MX}}}\left(1 - \exp\left(NTU\, \phi_{\mathrm{MX}}\right)\right)\right) \tag{15}$$

Note that the factor ϕ_{MX} is required in evaluating the local effectiveness to convert the conventional NTU_{MIN} to the required NTU_{UN} , or

$$NTU_{UN} = \frac{UA}{\dot{C}_{\mathrm{UN}}} = \frac{UA}{\dot{C}_{\mathrm{MX}}} \frac{\dot{C}_{\mathrm{MX}}}{\dot{C}_{\mathrm{UN}}} = NTU\, \phi_{\mathrm{MX}} \tag{16}$$

On the other hand, when the unmixed fluid is the minimum fluid, the ratio P_{MX} is not the effectiveness, so it must be adjusted while the NTU_{UN} is the correct NTU_{MIN} . The correct adjustment is simply

$$\varepsilon_{\mathrm{XF}} = \frac{\dot{C}_{\mathrm{MX}} \Delta T_{\mathrm{MX}}}{\dot{C}_{\mathrm{UN}} \Delta T_{\mathrm{MAX}}} = \frac{\Delta T_{\mathrm{MX}}}{\Delta T_{\mathrm{MAX}}} \frac{\dot{C}_{\mathrm{MX}}}{\dot{C}_{\mathrm{UN}}} = P_{\mathrm{MX}} \phi_{\mathrm{MX}} \tag{17}$$

Consequently, when the unmixed fluid is the minimum fluid and $\phi_{\mathrm{MX}} > 1.0$,

$$\varepsilon_{\mathrm{XF}} = P_{\mathrm{MX}} \frac{\dot{C}_{\mathrm{MX}}}{\dot{C}_{\mathrm{UN}}} = \phi_{\mathrm{MX}} \left\{1 - \exp\left(-\frac{1}{\phi_{\mathrm{MX}}}\left(1 - \exp\left(NTU\right)\right)\right)\right\} \tag{18}$$

The familiar plot of effectiveness versus NTU (specifically NTU_{min}) with heat capacity rate ratio as a parameter calculated from the preceding formulas is presented as Figure 2. Students will probably be gratified to realize several facts: (1) This fairly complicated result has a relatively simple basis, (3) The familiar equation for the effectiveness when one temperature is fixed is not universal, but it does have multiple applications, and (3) The effectiveness for this important case is relatively easy to compute and use in design applications. This figure and the other graphs in this paper were generated by an Excel spreadsheet that is available on request from the author or from the author's academic web site.

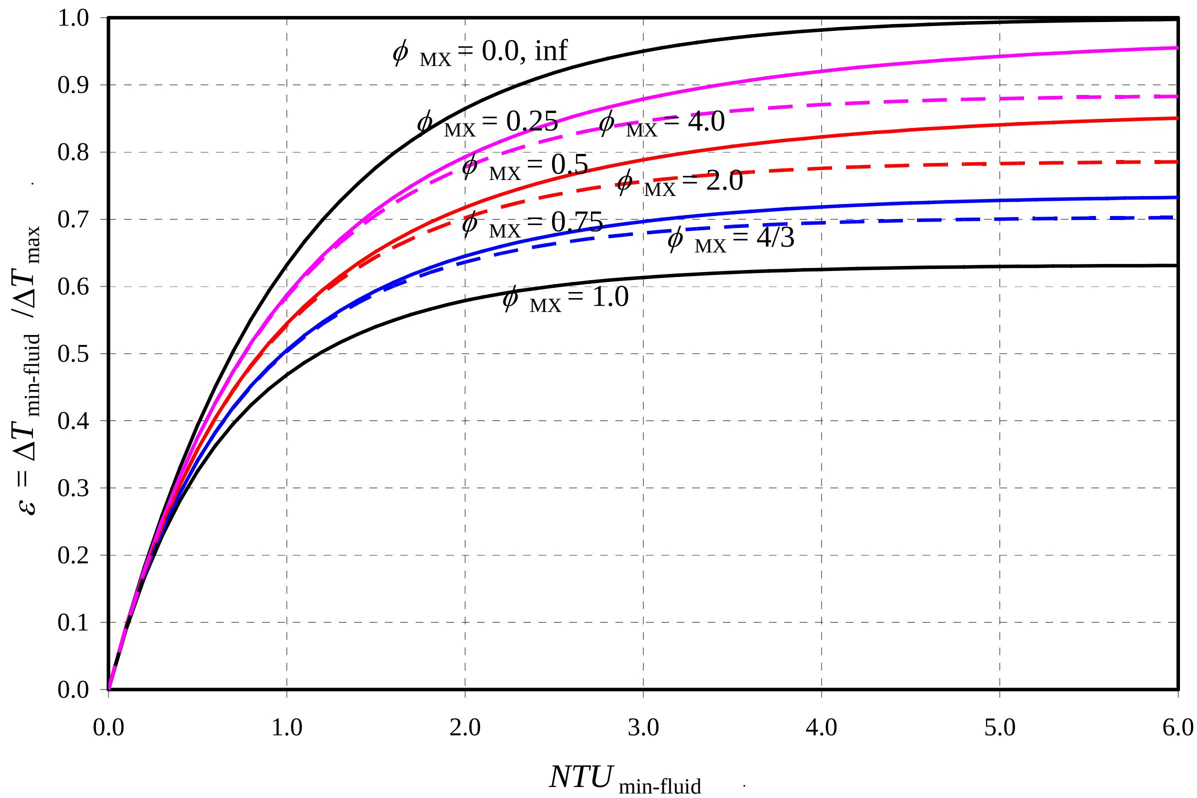

Figure 2. Effectiveness Results for a Cross Flow Exchanger when One Fluid Mixed and One Unmixed

ANALYSIS BY THE MEAN TEMPERATURE METHOD

The next analysis is conducted using the mean temperature method. This method seems to be somewhat obsolete; nevertheless, it seems to offer some educational attractions since it is concise and very concrete in its approach. In any event, it is at least of historical significance. The following analysis is probably somewhat original if only to show the relationships among the various approaches. The final result is in agreement with the graphical results presented long ago by Smith (1934) even though the original appears to have a typo error in the published equation.

The mean temperature is defined for the overall heat transfer rate by the simple expression

$$\dot{Q}_{\mathrm{ex}} = U\,A\Delta T_{\mathrm{mean}} \tag{19}$$

To proceed with the mean temperature method, next define a mean temperature difference ratio, R_{mean}, such that

$$R_{\mathrm{mean}} = \frac{\Delta T_{\mathrm{mean}}}{\Delta T_{\mathrm{MAX}}} \tag{20}$$

Later on it will be necessary to somehow eliminate the *UA* product from an effectiveness formula. Being aware of the previous findings, it is chosen to accomplish this task by writing the heat rate in terms of the mean temperature difference and equivalently in terms of the change of the temperature of the unmixed fluid, or

$$\dot{Q}_{\mathrm{ex}} = U\,A\Delta T_{\mathrm{mean}} = U\,A\,R_{\mathrm{mean}}\Delta T_{\mathrm{MAX}} = \dot{C}_{\mathrm{UN}}\;P_{\mathrm{UN}}\Delta T_{\mathrm{MAX}} \tag{21}$$

Then obviously,

$$\frac{U\,A}{\dot{C}_{\mathrm{UN}}} = \frac{P_{\mathrm{UN}}}{R_{\mathrm{mean}}} \tag{22}$$

Also note from energy conservation,

$$\dot{Q}_{\mathrm{ex}} = \dot{C}_{\mathrm{MX}}\;\Delta T_{\mathrm{MX}} = \dot{C}_{\mathrm{UN}}\;\Delta T_{\mathrm{UN}}\;,$$

so

$$\frac{\dot{C}_{\mathrm{UN}}}{\dot{C}_{\mathrm{MX}}} = \frac{\Delta T_{\mathrm{MX}}}{\Delta T_{\mathrm{UN}}} = \frac{P_{\mathrm{MX}}}{P_{\mathrm{UN}}} \tag{23}$$

Again, with knowledge of the preceding, it is next chosen to rewrite the equation for the mixed temperature ratio in terms of the unmixed ratio and the mean temperature ratio, so Equation (11) repeated here as

$$P_{\mathrm{MX}} = 1 - \exp\left(-\frac{\dot{C}_{\mathrm{UN}}}{\dot{C}_{\mathrm{MX}}}\,\varepsilon_{\mathrm{L}}\right)$$

is rewritten using Equation (22) in the ε_{L} function and Equation (23) to eliminate the heat capacity rate ratio as

$$P_{\mathrm{MX}} = 1 - \exp\left(-\frac{P_{\mathrm{MX}}}{P_{\mathrm{UN}}}\left(1 - \exp\left(-\frac{P_{\mathrm{UN}}}{R_{\mathrm{mean}}}\right)\right)\right) \tag{24}$$

Then it's fairly easy to solve for R_{mea} giving, in the form presented in the original publication (Smith, 1934).

$$R_{\mathrm{mean}} = \frac{P_{\mathrm{UN}}}{\ln\left(\left(1 - \frac{P_{\mathrm{UN}}}{P_{\mathrm{MX}}}\ln\left(\frac{1}{1 - P_{\mathrm{MX}}}\right)\right)^{-1}\right)} \tag{25}$$

The original paper has an obvious typo error in this equation. The results of this analysis are presented in Figure 3 below. It is notable that this method is concise and concrete. Note that the mean temperature is maximized when the temperature change of either fluid is minimized, which is obvious upon reflection, and it decreases monotonically as either temperature change increases. The mean temperature formulation does seem useful and simple as its parameters are simple and only one family of curves is necessary to present it. Indeed it should be kept in mind for further consideration. Nevertheless, it does not seem to have demonstrated the intellectual content of either the effectiveness or LMTD formulations.

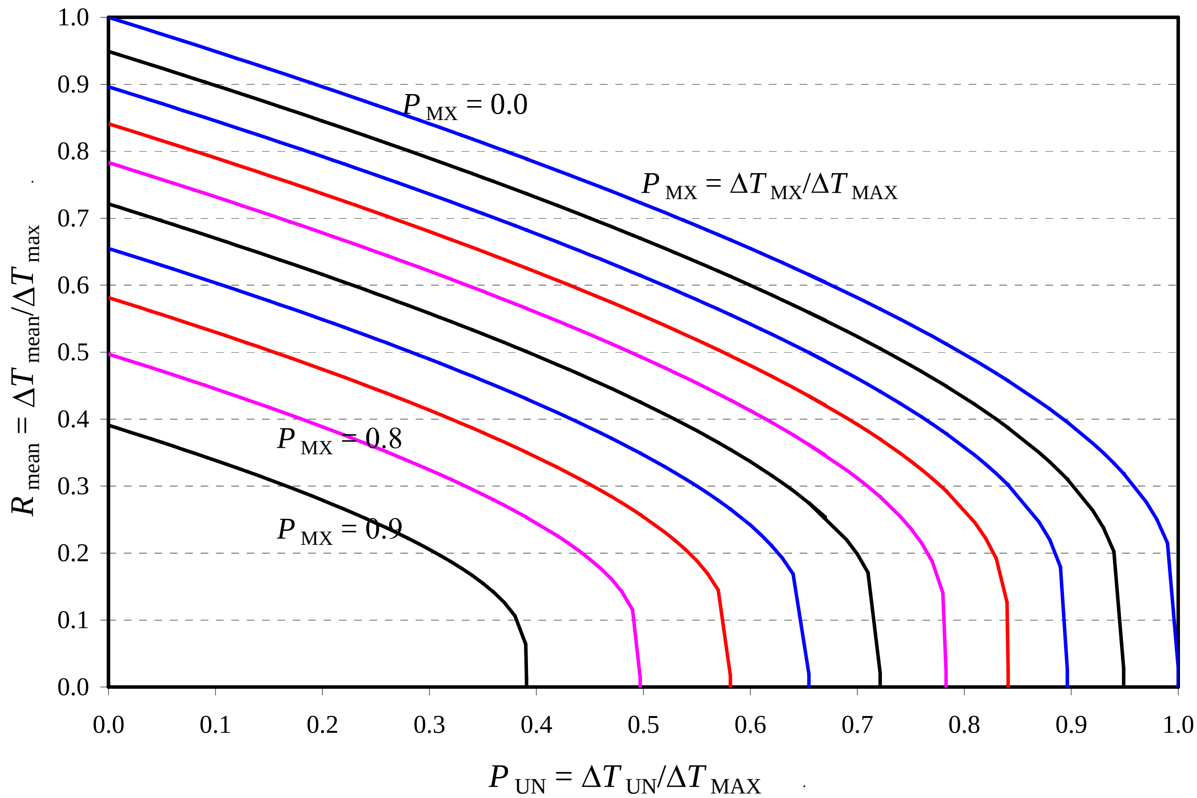

Figure 3. Mean Temperature Difference for Heat transfer in a Cross Flow Exchanger with only One Fluid Mixed

ANALYSIS BY THE LMTD METHOD

The next analysis is conducted using the LMTD method. This method seems to be as widely used as the effectiveness method. Indeed it seems to be favored for heat exchanger sizing whereas the effectiveness method seems to be favored for performance analysis or simulation. The approach favored by this author is to manipulate the effectiveness data finally resulting in LMTD and F_G data. Nevertheless, it is recognized that the mean temperature results could be an equivalent starting point.

Begin by writing the heat rate in terms of the LMTD.

$$\dot{Q}_{ex} = F_G (UA) LMTD \tag{26}$$

where F_G is the geometric correction factor, which is unique and representative of every thermally distinct design, and *LMTD* is the generic Log Mean Temperature Difference, which in this case is written as

$$LMTD = \frac{(T_{MX,,in} - T_{UN,out}) - (T_{MX,out} - T_{UN,in})}{\ln\left(\frac{(T_{MX,,in} - T_{UN,out})}{(T_{MX,out} - T_{UN,in})}\right)} \tag{27}$$

It is conventional to express the results of this analysis in terms of two temperature difference ratios. One is the P_{UN} already introduced and the other, here called R_{HC}, is actually the reciprocal of the heat capacity rate ratio already introduced in Equation (13), so the LMTD will be rewritten in terms of these variables as

$$LMTD = \frac{\left(\frac{1}{R_{\mathrm{HC}}} - 1\right)}{\ln\left(\frac{(1 - R_{\mathrm{HC}}\ P_{\mathrm{UN}})}{(1 - P_{\mathrm{UN}})}\right)}\left(T_{\mathrm{MX,,in}} - T_{\mathrm{MX,out}}\right) = \frac{\left(\frac{1}{R_{\mathrm{HC}}} - 1\right)}{\ln\left(\frac{(1 - R_{\mathrm{HC}}\ P_{\mathrm{UN}})}{(1 - P_{\mathrm{UN}})}\right)}\Delta T_{\mathrm{MX}} \tag{28}$$

Notice that the LMTD has now been expressed as a function of the temperature change for the mixed fluid. The temperature difference ratio R_{HC} used in Equation (28) is

$$R_{\mathrm{HC}} = \frac{T_{\mathrm{MX,,in}} - T_{\mathrm{MX,out}}}{T_{\mathrm{UN,out}} - T_{\mathrm{UN,in}}} = \frac{\dot{C}_{\mathrm{UN}}}{\dot{C}_{\mathrm{MX}}} = \frac{1}{\phi_{\mathrm{MX}}}$$

To evaluate the desired F_{G}, first introduce the heat rate for the mixed fluid into the defining Equation (26), which has been solved for F_{G}, so

$$F_{\mathrm{G}} = \frac{\dot{Q}_{\mathrm{ex}}}{(UA)LMTD} = \frac{\dot{C}_{\mathrm{MX}}\ \Delta T_{\mathrm{MX}}}{(UA)LMTD} = \frac{\Delta T_{\mathrm{MX}}}{(NTU_{\mathrm{MX}})LMTD} \tag{29}$$

Since the LMTD has already been written in terms of the temperature change for the mixed fluid, one may write

$$F_{\mathrm{G}} = \frac{\dot{Q}_{\mathrm{ex}}}{(UA)LMTD} = \frac{\dot{C}_{\mathrm{MX}}\ \Delta T_{\mathrm{MX}}}{(UA)LMTD} = \frac{\ln\left(\frac{(1 - R_{\mathrm{HC}}\ P_{\mathrm{UN}})}{(1 - P_{\mathrm{UN}})}\right)}{(NTU_{\mathrm{MX}})\left(\frac{1}{R_{\mathrm{HC}}} - 1\right)} \tag{30}$$

The preceding has the odd appearance of being a general solution for the F_{G}; however, NTU_{MX} or equivalent data are required to apply this equation to a particular geometry. In this case it is first necessary to solve Equation (10) for

$$\varepsilon_{\mathrm{L}} = -\left(\ln(1 - P_{\mathrm{UN}} R_{\mathrm{HC}})\right)/R_{\mathrm{HC}} \tag{31}$$

Then the required NTU_{MX} is

$$NTU_{\mathrm{MX}} = -R_{\mathrm{HC}} \ln(1 - \varepsilon_{\mathrm{L}}) \tag{32}$$

The F_{G} can then be plotted in the usual fashion as shown in Figure 4. This familiar figure was produced by an Excel workbook available from the author.

It is obviously desirable to present both the effectiveness and LMTD methods. Both methods are needed since most practitioners and students find the effectiveness method more convenient for performance analysis and simulation, while the LMTD method seems handy for simple sizing.

CLOSURE

As seen above, analysis by either of three distinct methods can develop performance and sizing tools for the cross flow exchanger and other types. The analysis of the cross flow exchanger is straightforward but worthy of classroom time particularly since the design and analysis charts can easily be misinterpreted. The most common results are the effectiveness and NTU methods. These methods are seen to be equivalent. Indeed in this paper, the LMTD presentation is derived from the effectiveness results. The mean temperature method was also exhibited. This method seems to be nearly obsolete, but it could be useful perhaps for summarizing numerical results.

Combined with conventional presentations for shell and tube exchangers, a presentation such as the one developed herein should fit the needs of most undergraduates. Indeed it has been successfully presented during two one-semester energy systems courses.

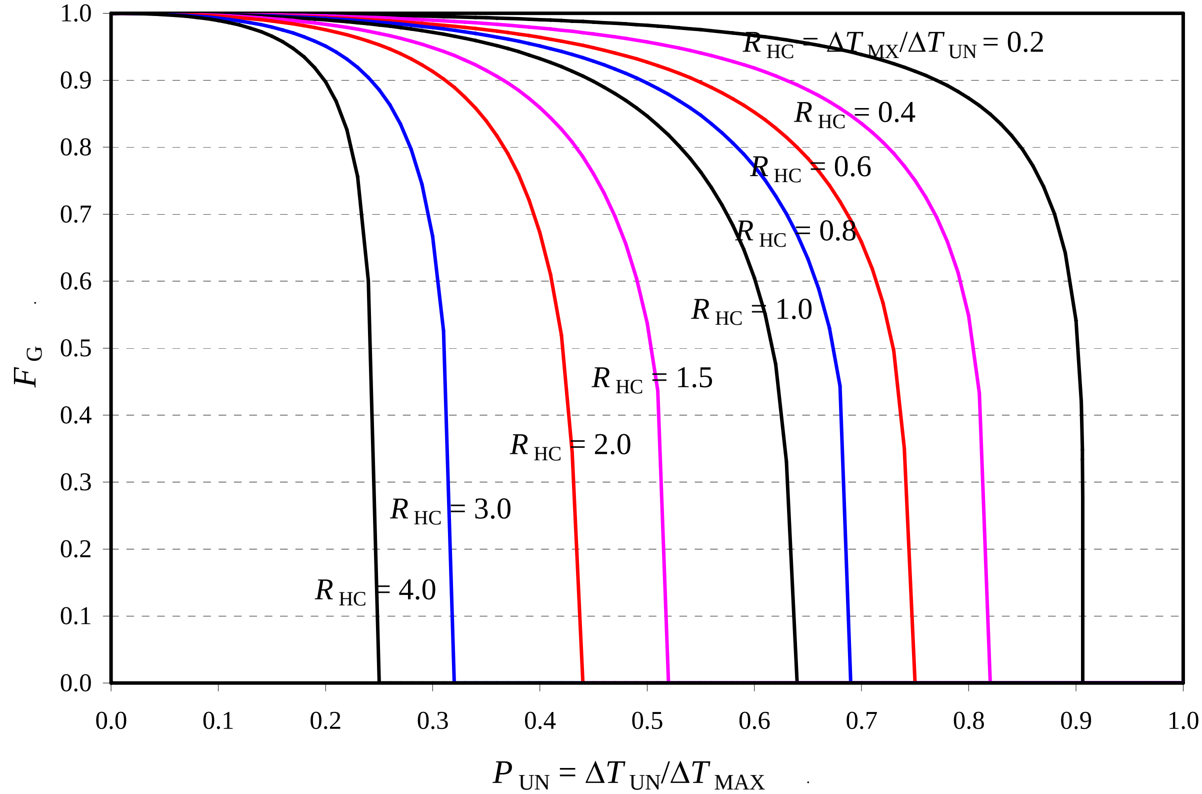

Figure 4. Geometric Correction Factor for Heat transfer in a Cross Flow Exchanger with only one Fluid Mixed
MX refers to mixed fluid, and UN refers to unmixed fluid.

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Sheldon M. Jeter

Sheldon Jeter is Associate Professor of Mechanical Engineering at the George W. Woodruff School of Mechanical Engineering at Georgia Tech. He has degrees from Clemson University, the University of Florida, and Georgia Tech. He has been on the academic faculty at Georgia Tech since 1979. His research interests are thermodynamics, experimental engineering, heat and mass transfer, solar energy, and general energy systems.