Two-Dimensional Temperature Distribution in a Transformer Core-and-Coil Assembly with Heat Generation

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Abstract

A residential distribution transformer consists of a cylindrical steel tank and a core-and-coil assembly placed concentrically within the tank. The tank is nearly filled with cooling fluid, so that the heat generated inside the core-and-coil can be dissipated to the surrounding air through the cooling fluid. The transfer of heat is mostly in the radial direction through natural convection of the fluid. The power rating of the transformer can be from 10 to 100 kVA, with the heat loss at full load operation ranging from 178 to 1,177 watts. The diameter of the core-and-coil can be from 29 to 47 cm, and its height from 23 to 51 cm. The present study is on the mathematical and numerical analyses of the temperature variation within the core-and-coil assembly. Due to the continuous flow of electrical current in the coil winding, and due primarily to the electrical resistance of the winding material, a small fraction of the electrical energy is converted into heat energy, and the heat energy is physically exhibited in the form of an elevated temperature in the winding. To study the operational characteristics of the device, especially to prevent the malfunction or even the failure of the device, it is important to know the temperature variation within the device. A previous study [1] based on the one-dimensional analysis of the core-and-coil assembly operating under steady state condition shows that the factors to be considered are: the thermal conductivity of the core-and-coil assembly, the amount of heat generating within the electrical conductor, and the physical dimensions of the core-and-coil assembly.

In the present study, the two-dimensional analysis is to be taken. The variation of temperature with respect to the axial position as well as the radial direction is to be considered. This approach indicates that a partial differential equation is to be solved. The equation consists of the temperature as the dependent variable, and the geometric coordinates in the axial direction and the radial position as the independent variables. The parameters contained in the original equation are the amount of heat generation, the thermal conductivity of the core-and-coil assembly, and, through the boundary conditions, the over-all height, the inner and outer radii, and the surface temperatures of the assembly.

All engineering students acquire their proficiency in mathematics through the required courses in calculus, differential equations, computer programming, and engineering mathematics. Some engineering majors need additional math courses such as linear algebra, advanced calculus, complex variables, or partial differential equations. For an engineering problem which requires the solution of a partial differential equation, such as in the present case, a packaged computer program using finite element analysis can provide a graphical representation of the solution. However, in order to see the beauty and the power of science and mathematics, it is probably best to express the

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solutions in a mathematical form, and then to extract the necessary numerical values from these final equations.

Introduction

A group of electromagnetic devices, which employ the principle of mutual induction to convert variations of current in a primary circuit into variations of voltage and current in a secondary circuit, such as a transformer, an electromagnetic relay, or a generator, consists of an electrical coil which is wound around a core made of either an iron, a steel, or a cobalt. The core is usually of cylindrical in configuration, hence the complete assembly is also of cylindrical shape. A graphical representation of a residential distribution transformer is shown in Figure 1. The core and coil assembly is also shown in graphical form in the same figure. When electrical current flows through the coil, a small percentage of the electrical energy is lost in the form of heat. These losses consist of resistance loss in the windings, hysteresis loss in the core, and eddy currents in the core. The total loss is usually less than 10% of the total energy supplied. Due to these energy losses, the temperature in the assembly increases in comparison with its ambient, and if sufficient provision is provided for the continuous dissipation of heat, a steady state temperature distribution will eventually be reached.

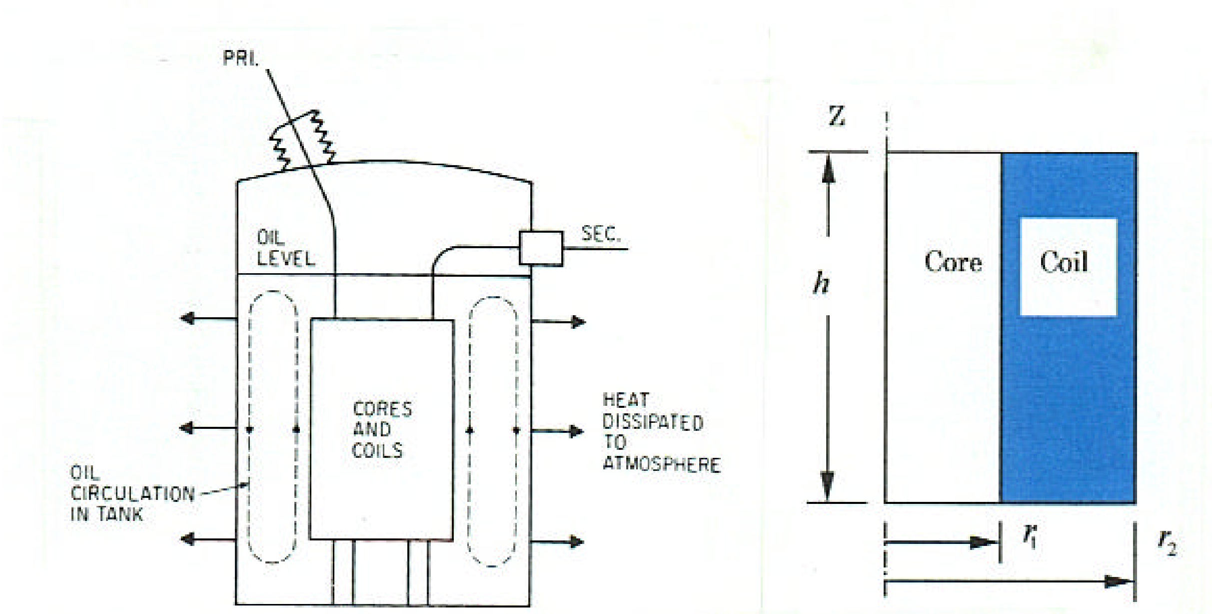

Figure 1. Graphical Representations of a Residential Distribution Transformer and the Core and Coil Assembly

Two Dimensional Analysis

During the steady state operation of an electromagnetic device, heat is generated within the coil assembly. Since the coil carries electrical current, and the transmission of an electrical current is an irreversible process, a small fraction of the electrical energy is converted into heat energy, which is physically represented by the increase in temperature. The rate of heat generation in joules per cubic meter per second, represented by q_e, is given by the following expression [2], [3] :

$$q_e = r_e I^2 \qquad (J/s - m^3 \ or \ W/m^3) \tag{1}$$

where r_e is the electrical resistivity of the winding, in $ohm-m$, and I is the current density, in A/m^2.

As for the core of the device, there are magnetic flux lines passing through the magnetic material. However, since the flux lines represent the alteration of the atomic structure of the core material due to external force (i.e., the magneto-motive force), and is not a measure of the flow of charged particles through the core material, therefore, there is no energy generation or loss within the core. The temperature of the core remains essentially unchanged once a steady state is reached.

In a realistic situation, the variation of temperature in both the radial and the axial directions should be considered. The governing differential equation for such a case is given by the following relation [3], [5]:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}+\frac{q_e}{k}=0 \qquad (2)$$

Where k is the thermal conductivity of the core-and-coil assembly, in W/m-C, T is the temperature, in ^oC, r is the radial coordinate, in m, and z is the axial coordinate, in m.

To solve this partial differential equation, let a new dependent variable q be introduced such that the following relationship is true:

$$q=T-T_2-\frac{q_e}{2k}z(h-z) \quad or \quad T=q+T_2+\frac{q_e}{2k}z(h-z) \qquad (3)$$

In Equation (3), the term $q_e z(h-z)/2k$ represents the particular solution of the differential equation given in Equation (2), and T_2 is the temperature at the outer surface of the coil assembly. T_2 enters into the problem through the boundary conditions which will be introduced later. Then, Equation (2) can be transformed into the following equation:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial q}{\partial r})+\frac{\partial^2 q}{\partial z^2}=0 \qquad (4)$$

Based on the separation of variables technique, which is applied very often in the solution of partial differential equation, the solution of q can be represented by the product of two parts, namely, q_1 and q_2, where q_1 is a function of r only, and q_2 is a function of z only. Therefore, the following relationship can be written:

$$q=q_1(r)q_2(z) \qquad (5)$$

In terms of $q_1(r)$ and $q_2(z)$, Equation (4) becomes the following:

$$\frac{q_2}{r}\frac{d}{dr}(r\frac{dq_1}{dr})+q_1\frac{d^2 q_2}{dz^2}=0 \qquad (6)$$

The last equation can be rearranged into the following form:

$$\frac{1}{rq_1}\frac{d}{dr}(r\frac{dq_1}{dr}) = -\frac{1}{q_2}\frac{d^2q_2}{dz^2} = \pm m^2 \tag{7}$$

where m is a positive integer constant. It is called the separation constant. Notice that Equation (7) consists of two ordinary differential equations, one of which, $q_1(r)$, contains r as the independent variable, the other one, $q_2(z)$, contains z as the independent variable. Based on the regular procedure for the solution of a differential equation, the solution for $q_2(z)$ is given by either of the following equations:

$$\textit{For positive sign}:\quad q_2(z) = C_1\sin(mz) + C_2\cos(mz) \tag{8a}$$

$$\textit{For negative sign}:\quad q_2(z) = C_3\cosh(mz) + C_4\sinh(mz) \tag{8b}$$

Where C_1, C_2, C_3 and C_4 are the integration constants. Depending on the sign of the right side of Equation (7), two sets of possible solution can be obtained for $q_1(r)$. These are given below:

$$\textit{For positive sign}:\quad q_1(r) = C_5\, I_0(mr) + C_6\, K_0(mr) \tag{9a}$$

$$\textit{For negative sign}:\quad q_1(r) = C_7\, J_0(mr) + C_8\, Y_0(mr) \tag{9b}$$

Where C_5, C_6, C_7 and C_8 are all constants of integration. $I_0(mr)$ is the zero-order modified Bessel function of the first kind, $K_0(mr)$ is the zero-order modified Bessel function of the second kind, $J_0(mr)$ is the zero-order Bessel function of the first kind, and $Y_0(mr)$ is the zero-order Bessel function of the second kind [4]. The Bessel Functions of the first kind and the second kinds have been studied in a previous publication by the present author [1]. The modified Bessel functions will be studied in the following. In general, the products of modified Bessel functions with trigonometric functions are used as the solution when the temperature is prescribed as an arbitrary function on a plane boundary, and the products of Bessel functions with hyperbolic functions are used as the solution when the temperature is prescribed as an arbitrary function on a circular boundary.

In the present study, the solution of q in Equation (4) can be written as follows:

$$q = [C_1\sin(mz) + C_2\cos(mz)]\cdot[C_5\, I_0(mr) + C_6\, K_0(mr)] \tag{10}$$

The zero-order modified Bessel function of the first kind, $I_o(mr)$, and the zero-order modified Bessel function of the second kind, $K_o(mr)$, using r instead of mr as the argument, are given respectively as follows [4]:

$$I_0(r) = 1 + \frac{1}{(1!)^2}(\frac{r}{2})^2 + \frac{1}{(2!)^2}(\frac{r}{2})^4 + \frac{1}{(3!)^2}(\frac{r}{2})^6 + \frac{1}{(4!)^2}(\frac{r}{2})^8 + \cdots = \sum_{n=0}^{\infty}\frac{1}{(n!)^2}\cdot(\frac{r}{2})^{2n} \tag{11}$$

$$K_0(r) = -I_0(r)[\ell n(\frac{r}{2}) + 0.5772157] + \frac{1}{(1!)^2}(\frac{r}{2})^2 + \frac{1}{(2!)^2}(\frac{r}{2})^4(1+\frac{1}{2}) + \frac{1}{(3!)^2}(\frac{r}{2})^6(1+\frac{1}{2}+\frac{1}{3}) +$$

$$+\frac{1}{(4!)^2}(\frac{r}{2})^8(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}) + \cdots \qquad (12)$$

The boundary conditions to be applied to Equation (10) are listed below:

At $r = r_1$ *and* $0 < z < h$, *the net heat transfer is zero*

At $r = r_2$ *and* $0 < z < h$, *the temperature is* $T = T_2$, *or* $q = q_2 = 0$

At $z = 0$ *and* $r_1 < r < r_2$, *the temperature is* $T = T_2$, *or* $q = q_2 = 0$

At $z = h$ *and* $r_1 < r < r_2$, *the temperature is* $T = T_2$, *or* $q = q_2 = 0$

When these conditions are applied to Equation (10), the final solution is given below:

$$q = T - T_2 - \frac{q_e}{2k} \cdot z \cdot (h - z) = -\frac{4q_e h^2}{kp^3} \sum_{n=0}^{\infty} \frac{I_1(gr_1)K_0(gr) + K_1(gr_1)I_0(gr)}{I_1(gr_1)K_0(gr_2) + K_1(gr_1)I_0(gr_2)} \cdot \frac{\sin(gz)}{(2n+1)^3} \qquad (13)$$

$$where \quad g = \frac{p(2n+1)}{h}$$

In Equation (13), $I_1(x)$ *and* $K_1(x)$ are the first-order modified Bessel function of the first kind and the first-order modified Bessel function of the second kind, respectively. These are shown in Figure 2. For the purpose of mathematical completeness, higher order of modified Bessel functions are given in the next section.

General Forms of Bessel Function

A FORTRAN computer program has been written for the computation of the zero, first, second, and third order of the modified Bessel functions. The numerical results are shown in Table 1. The corresponding graphical representation is shown in Figure 2. In general, as the argument of the functions increases from zero, the values of the functions increase rapidly for the modified Bessel function of the first kind, $I_i(r)$, while the values decrease rapidly for the modified Bessel function of the second kind, $K_i(r)$.

The modified Bessel function of the first kind of order i can be expressed in a general form as follows [5], [8]:

$$I_i(r) = \sum_{n=0}^{\infty} \frac{(\frac{r}{2})^{i+2n}}{n!(i+n)!} \quad i = 0, 1, 2, 3, \cdots \qquad (14)$$

The modified Bessel function of the second kind of order i can be expressed in a general form as follows [5]:

Table 1. Modified Bessel Functions of the first kind (I) and second kind (K)

R	I0	I1	I2	I3	K0	K1	K2	K3
.00	1.00000	.00000	.00000	.00000	99999.0	99999.0	99999.0	9999999.0
.01	1.00003	.00500	.00001	.00000	4.72124	99.97390	19999.50000	7999901.00000
.02	1.00010	.01000	.00005	.00000	4.02846	49.95472	4999.50000	999950.10000
.03	1.00022	.01500	.00011	.00000	3.62353	33.27149	2221.72300	296263.00000
.04	1.00040	.02000	.00020	.00000	3.33654	24.92329	1249.50100	124975.00000
.05	1.00063	.02501	.00031	.00000	3.11423	19.90968	799.50130	63980.02000
.10	1.00250	.05006	.00125	.00002	2.42707	9.85385	199.50400	7990.01400
.15	1.00563	.07521	.00282	.00007	2.03003	6.47750	88.39669	2363.72300
.20	1.01003	.10050	.00502	.00017	1.75270	4.77597	49.51244	995.02470
.25	1.01569	.12598	.00785	.00033	1.54151	3.74703	31.51772	508.03050
.30	1.02263	.15169	.01133	.00057	1.37246	3.05599	21.74574	292.99920
.35	1.03086	.17769	.01547	.00090	1.23271	2.55912	15.85627	183.77370
.40	1.04040	.20403	.02027	.00135	1.11453	2.18435	12.03630	122.54740
.45	1.05127	.23074	.02574	.00192	1.01291	1.89152	9.41968	85.62198
.50	1.06348	.25789	.03191	.00265	.92442	1.65644	7.55018	62.05791
.60	1.09205	.31370	.04637	.00460	.77752	1.30283	5.12030	35.43820
.70	1.12630	.37188	.06379	.00737	.66052	1.05028	3.66133	21.97216
.80	1.16651	.43286	.08435	.01110	.56535	.86178	2.71980	14.46078
.90	1.21299	.49713	.10826	.01597	.48673	.71653	2.07903	9.95665
1.00	1.26607	.56516	.13575	.02217	.42102	.60191	1.62484	7.10126
1.10	1.32616	.63749	.16709	.02989	.36560	.50976	1.29244	5.20954
1.20	1.39373	.71468	.20260	.03936	.31851	.43459	1.04283	3.91069
1.30	1.46928	.79733	.24262	.05081	.27825	.37255	.85140	2.99223
1.40	1.55340	.88609	.28755	.06452	.24365	.32084	.70199	2.32653
1.50	1.64672	.98167	.33783	.08077	.21381	.27739	.58366	1.83380
1.60	1.74998	1.08481	.39397	.09989	.18795	.24063	.48875	1.46250
1.70	1.86397	1.19635	.45650	.12223	.16550	.20936	.41180	1.17832
1.80	1.98956	1.31717	.52604	.14819	.14593	.18262	.34885	.95784
1.90	2.12774	1.44824	.60327	.17820	.12885	.15966	.29691	.78473
2.00	2.27959	1.59064	.68895	.21274	.11389	.13987	.25376	.64739
2.10	2.44628	1.74550	.78390	.25235	.10078	.12275	.21768	.53738
2.20	2.62914	1.91409	.88906	.29763	.08927	.10790	.18736	.44855
2.30	2.82961	2.09780	1.00543	.34922	.07914	.09498	.16173	.37626
2.40	3.04926	2.29812	1.13415	.40787	.07022	.08372	.13999	.31704
2.50	3.28984	2.51672	1.27647	.47437	.06235	.07389	.12146	.26823
2.60	3.55327	2.75538	1.43374	.54963	.05540	.06528	.10562	.22777
2.70	3.84165	3.01611	1.60750	.63463	.04926	.05774	.09202	.19407
2.80	4.15730	3.30105	1.79940	.73048	.04382	.05111	.08033	.16587
2.90	4.50275	3.61261	2.01129	.83841	.03901	.04529	.07024	.14217
3.00	4.88079	3.95337	2.24521	.95975	.03474	.04016	.06151	.12217
3.10	5.29449	4.32620	2.50339	1.09602	.03096	.03563	.05394	.10524
3.20	5.74720	4.73425	2.78830	1.24888	.02759	.03164	.04737	.09086
3.30	6.24263	5.18095	3.10265	1.42016	.02461	.02812	.04165	.07860
3.40	6.78481	5.67010	3.44945	1.61191	.02196	.02500	.03666	.06813
3.50	7.37820	6.20583	3.83201	1.82639	.01960	.02224	.03231	.05916
3.60	8.02768	6.79271	4.25395	2.06610	.01750	.01980	.02850	.05146
3.70	8.73861	7.43574	4.71929	2.33380	.01563	.01763	.02516	.04483
3.80	9.51688	8.14041	5.23245	2.63257	.01396	.01571	.02223	.03911
3.90	10.36894	8.91277	5.79829	2.96581	.01249	.01400	.01966	.03416
4.00	11.30190	9.75945	6.42218	3.33727	.01116	.01248	.01740	.02989
4.10	12.32355	10.68773	7.11003	3.75111	.00998	.01114	.01541	.02617
4.20	13.44244	11.70560	7.86834	4.21194	.00892	.00994	.01366	.02295
4.30	14.66795	12.82187	8.70429	4.72486	.00799	.00887	.01211	.02014
4.40	16.01041	14.04620	9.62577	5.29549	.00715	.00792	.01075	.01770
4.50	17.48114	15.38920	10.64150	5.93008	.00640	.00708	.00955	.01556
4.60	19.09259	16.86253	11.76105	6.63553	.00573	.00632	.00849	.01370
4.70	20.85842	18.47904	12.99499	7.41946	.00513	.00565	.00754	.01207
4.80	22.79363	20.25279	14.35496	8.29032	.00459	.00505	.00670	.01064
4.90	24.91473	22.19930	15.85378	9.25743	.00412	.00453	.00596	.00939
5.00	27.23982	24.33559	17.50558	10.33112	.00369	.00405	.00531	.00829

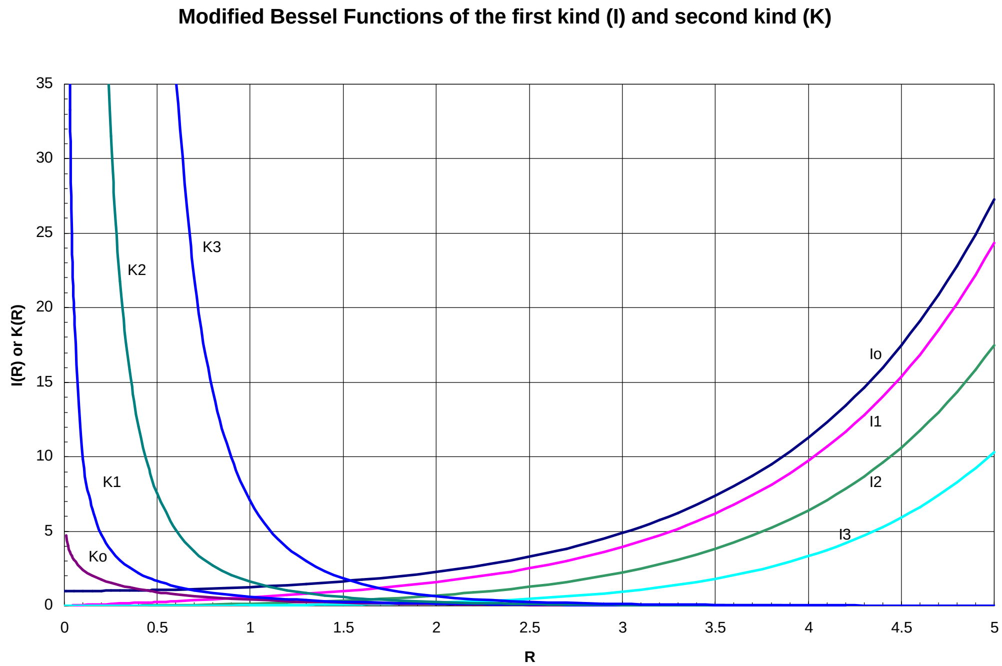

Figure 2. Modified Bessel Functions of the First Kind (I_i) and Second Kind (K_i)

$$K_i(r) = (-1)^{i+1} \cdot I_i(r)[\ln(\frac{r}{2}) + 0.5772157] + \frac{(-1)^i}{2} \sum_{n=0}^{\infty} \frac{(\frac{r}{2})^{i+2n}}{n!(i+n)!} [\sum_{m=1}^{i+n} m^{-1} + \sum_{m=1}^{n} m^{-1}] + $$
$$+ \frac{1}{2} \sum_{n=0}^{i-1} (-1)^n (\frac{r}{2})^{-i+2n} \cdot \frac{(i-n-1)!}{n!} \qquad (15)$$

$$\text{In Equation (15), for } n = 0, \text{ replace } \sum_{m=1}^{i+n} m^{-1} + \sum_{m=1}^{n} m^{-1} \quad \text{by} \quad \sum_{m=1}^{i} m^{-1}$$

The complete mathematical expressions for the first, second, and third order modified Bessel function are given in the following:

For i=1:

$$I_1(r) = \frac{1}{0!\cdot 1!}(\frac{r}{2}) + \frac{1}{1!\cdot 2!}(\frac{r}{2})^3 + \frac{1}{2!\cdot 3!}(\frac{r}{2})^5 + \frac{1}{3!\cdot 4!}(\frac{r}{2})^7 + \frac{1}{4!\cdot 5!}(\frac{r}{2})^9 + \cdots \qquad (16)$$

$$K_1(r) = I_1(r)[\ell n(\frac{r}{2}) + 0.5772157] - \frac{1}{2}[\frac{1}{0!\cdot 1!}(\frac{r}{2})^1(1) + \frac{1}{1!\cdot 2!}(\frac{r}{2})^3 \cdot (1+1+\frac{1}{2}) +$$

$$+\frac{1}{2!\cdot 3!}(\frac{r}{2})^5\cdot(1+\frac{1}{2}+1+\frac{1}{2}+\frac{1}{3})+\frac{1}{3!\cdot 4!}(\frac{r}{2})^7(1+\frac{1}{2}+\frac{1}{3}+1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4})+\cdots] \quad (17)$$

For i=2:

$$I_2(r)=\frac{1}{0!\cdot 2!}(\frac{r}{2})^2+\frac{1}{1!\cdot 3!}(\frac{r}{2})^4+\frac{1}{2!\cdot 4!}(\frac{r}{2})^6+\frac{1}{3!\cdot 5!}(\frac{r}{2})^8+\frac{1}{4!\cdot 6!}(\frac{r}{2})^{10}+\cdots \quad (18)$$

$$K_2(r)=-I_2(r)[\ell n(\frac{r}{2})+0.5772157]+\frac{1}{2}[(-1)^0(\frac{r}{2})^{-2}\cdot\frac{1!}{0!}+(-1)^1(\frac{r}{2})^0\frac{0!}{1!}]+\frac{1}{2}[\frac{1}{0!\cdot 2!}(\frac{r}{2})^2\cdot(0+1+\frac{1}{2})+$$

$$+\frac{1}{1!\cdot 3!}(\frac{r}{2})^4(1+1+\frac{1}{2}+\frac{1}{3})+\frac{1}{2!\cdot 4!}(\frac{r}{2})^6(1+\frac{1}{2}+1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4})+\frac{1}{3!5!}(\frac{r}{2})^8(1+\frac{1}{2}+\frac{1}{3}+$$

$$+1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5})+\cdots] \quad (19)$$

For i=3:

$$I_3(r)=\frac{1}{0!\cdot 3!}(\frac{r}{2})^3+\frac{1}{1!\cdot 4!}(\frac{r}{2})^5+\frac{1}{2!\cdot 5!}(\frac{r}{2})^7+\frac{1}{3!\cdot 6!}(\frac{r}{2})^9+\frac{1}{4!\cdot 7!}(\frac{r}{2})^{11}+\cdots \quad (20)$$

$$K_3(r)=I_3(r)[\ell n(\frac{r}{2})+0.5772157]+\frac{1}{2}[(\frac{r}{2})^{-3}\cdot\frac{2!}{0!}-(\frac{r}{2})^{-1}\cdot\frac{1!}{1!}+(\frac{r}{2})^{1}\frac{0!}{2!}]+$$

$$-\frac{1}{2}[\frac{1}{0!\cdot 3!}(\frac{r}{2})^3\cdot(1+\frac{1}{2}+\frac{1}{3})+\frac{1}{1!\cdot 4!}(\frac{r}{2})^5(1+1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4})+\frac{1}{2!\cdot 5!}(\frac{r}{2})^7(1+\frac{1}{2}+$$

$$+1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5})+\frac{1}{3!\cdot 6!}(\frac{r}{2})^9(1+\frac{1}{2}+\frac{1}{3}+1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6})+\cdots] \quad (21)$$

Temperature Computation

With the FORTRAN program for the numerical evaluation of the Bessel functions available at this point, a second FORTRAN program is prepared for the computation of the temperature variation in the core-and-coil assembly. In this program, the axial position, i.e., the z-coordinate, is divided into ten equal divisions, and for each axial position, the radial-coordinate, from the inner radius r_1 to the outer radius r_2, is divided into ten equal distances. A typical numerical output is shown in Table 2. Figure 3 is a graphical display of the temperature profiles. The parameter used in the diagram is the axial coordinate z. As it is expected physically, the temperature decreases from the inner surface of the core-and-coil assembly to its outer surface, where the heat is dissipated by the cooling fluid to the tank. Also, at a given radial location, the temperature is the highest at the mid-point of the axial direction, and it decreases gradually to the top and bottom surfaces.

```
K=  200.00(W/m-C)    QE= 4000000.(W/m**3)    T2=  50.00(C)    R1= .0900(m)
R2=  .1800(m)        Z=  .0000(m)            H=  .3000(m)     RA= .090000(m)

R=  .090000(m)   Z=  .00000(m)
        R (m)      T (C)
       .090000     50.00
       .099000     50.00
       .108000     50.00
       .117000     50.00
       .126000     50.00
       .135000     50.00
       .144000     50.00
       .153000     50.00
       .162000     50.00
       .171000     50.00
       .180000     50.00

R=  .090000(m)   Z=  .03000(m)
        R (m)      T (C)
       .090000     78.11
       .099000     77.84
       .108000     77.06
       .117000     75.79
       .126000     74.02
       .135000     71.74
       .144000     68.89
       .153000     65.42
       .162000     61.20
       .171000     56.14
       .180000     50.04
```

Table 2. Typical Output for Temperature Computation

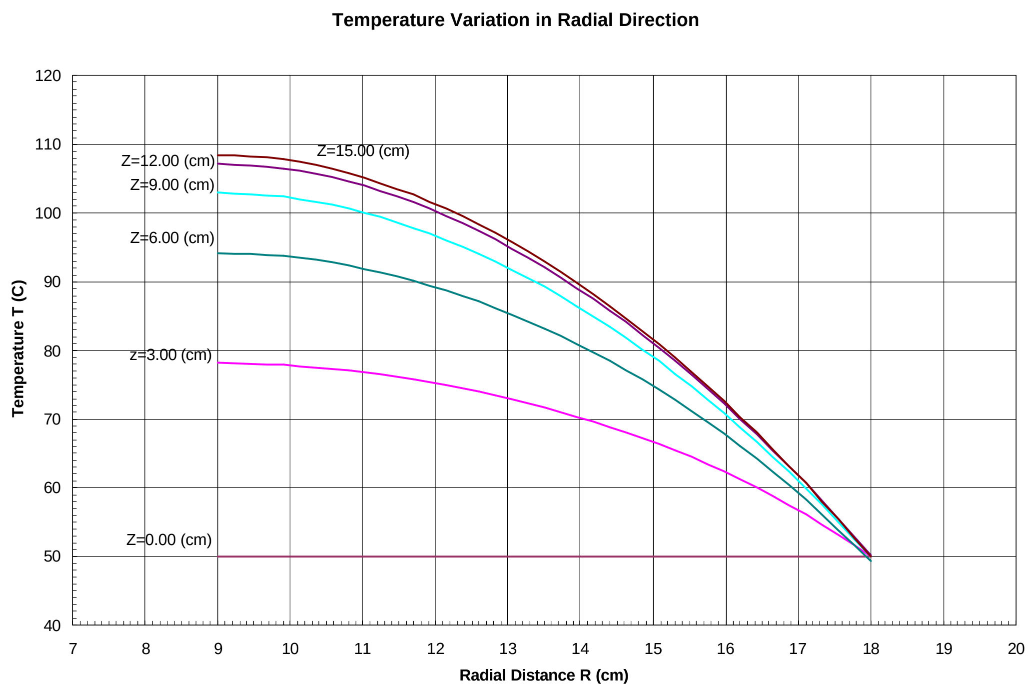

Figure 3. Temperature as a Function of Radial Coordinate, with Axial Coordinate as Parameter

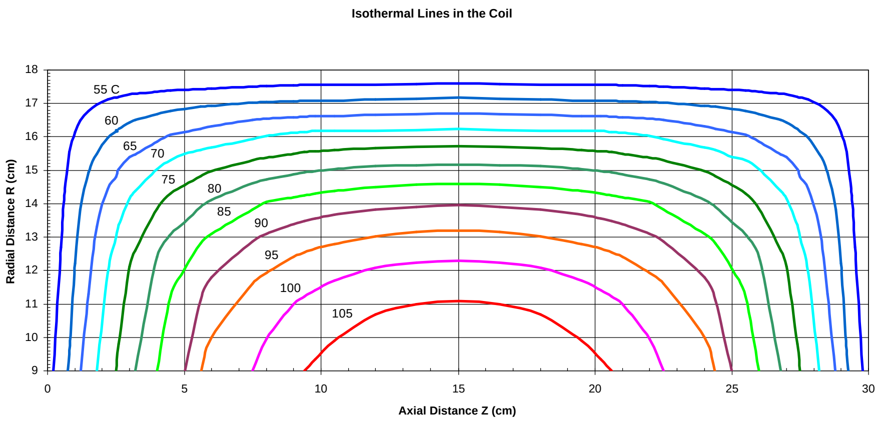

Figure 4. The Isothermal Lines in the Coil Structure

Figure 4 displays the isothermal lines in the coil structure. The boundary conditions specify the top, the bottom, and the outside surfaces to be all at a temperature of $50\,^{o}C$. From these surfaces, heat is carried away by the cooling oil through natural convection. The temperature is the highest at the axial center point (i.e., a circular line) of the inner surface, where the axial coordinate is Z=15.0 cm, and the radial coordinate is R=9.0 cm. The maximum temperature is T=108.47 ^{o}C. From this point, the temperature decreases both in the axial and radial directions.

Summary

On the study of the temperature variation in the core and coil of a distribution transformer, the two-dimensional analysis indicates that the mathematical solution can be expressed in terms of the product of infinite series and transcendental functions. The infinite series are the modified Bessel functions, and the transcendental functions are the trigonometric functions. Using computer programs developed in the present study, the numerical results can be obtained by the summation of successive terms in the solution, until a term is reached where the numerical value is very small, such as 1×10^{-6}, so that any following terms can be dropped. Within the cylindrical shell of the coil assembly, the numerical results show that the temperature is the highest at the axial center point (i.e., a circular line) of the inner surface. From this location, the temperature decreases both in the axial and radial directions.

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