A Unified Approach to Piping System Problems

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Abstract – A unified approach to the solution of series, parallel, and network piping system problems is investigated. Examples of series, parallel, and network piping system solutions using a unified solution methodology are presented and discussed. Solutions for all piping problems are formulated as a single nonlinear algebraic equation or as a system of nonlinear algebraic equations and a computational software system (Mathcad in this paper) is used for the arithmetic. This arrangement permits the student to concentrate on problem formulation and results (the engineering aspects of the problems) rather than on the arithmetic. The congruence of the problem formulations for all problems is evident to the student.

Keywords: Piping Systems, Piping Networks

INTRODUCTION

Many of the "procedures" for solving engineering problems are formulations to solve a non-linear algebraic equation or a system of non-linear algebraic equations. However, recent computational software systems, such a Mathcad and EES, have made possible "direct" solutions of such non-linear problems in which the solution procedure is transparent to the user. Piping systems are an excellent example of such problems. The purposes of this paper are twofold: (1) to explore the effects of the use of a computational software system for piping system problem solutions and (2) to investigate the pedagogical inferences of the use of such software in undergraduate engineering education involving piping system topics.

BACKGROUND

Most undergraduate courses in fluid mechanics address the flow of viscous fluids in pipes and develop techniques suitable for the solution of simple piping system problems. Piping systems are characterized as series, parallel, or network. Generally, piping systems with components in series are examined first and solutions are classed as Category I (find the increase in head of a pump), Category II (find the flow rate in a system), and Category III (find the appropriate pipe diameter, if it exists, for a given situation). Most first courses in fluid mechanics do not contain detailed coverage of parallel systems or fluid networks. In a first course, or in a follow-on fluid mechanics or thermal systems course, if solution techniques for parallel systems and fluid networks are covered, the solution "procedures" are associated with, but are considered distinct from, series systems. The advent of computational software systems (for example Mathcad, Mathematica, Matlab, and EES) permits a much more unified solution approach to all types of piping system problems. From a pedagogical standpoint, the unified approach permits the student to focus more on the engineering aspects than the arithmetic aspects, and from an applications standpoint, the unified approach provides the student with a useful addition to the student's engineering skill set.

No matter what the characterization (series, parallel, or network) of a piping system, the same fundamental principles are used in the unified solution formulation. The fundamental principles are delineated as follows: (1) conservation of mass, (2) conservation of energy, and (3) uniqueness of pressure at a point. The conventional solution "procedures" developed for any characterization of piping problem satisfy these principles either by formally invoking them as part of the problem formulation or by using them in a specified iterative sequence—the "procedure." Solutions for **all** series, parallel, and network piping problems can be formulated as a solution to a non-

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linear equation or to a system of nonlinear algebraic equations. The aforementioned computational software systems contain robust options for the solutions to systems of nonlinear algebraic equations. The computational system then becomes the arithmetic engine for the solution, and the student can concentrate on problem formulation and results (engineering aspects of the problems) rather than on the arithmetic. The results are better mastery of piping system problems, exposure to more realistic problems, and a graduate better equipped to handle meaningful piping problems. The congruence of the problem formulations for all problems is evident to the student.

The unified approach to piping systems uses the energy equation (Hodge and Taylor, 1999), cast between two stations in a pipe with a flowing fluid as a fundamental building block. Consider, as in Figure 1, the flow of an incompressible fluid through a segment of a pipe with an active device (pump or turbine) and major and minor losses. For this pipe segment with a pump, the energy equation becomes

$$\frac{P_A - P_B}{\gamma} = Z_B - Z_A + \frac{V^2}{2g}\left(f\frac{L}{D} + K + C \cdot f_T \right) - W_s \frac{g_c}{g} \qquad (1)$$

where W_s is the increase in head of the pump. Conservation of mass appears as

$$Q = Q_A = Q_B \qquad (2)$$

In Equation (1), expressions for the friction factor and fully-rough friction factor are needed. In introductory fluid mechanics courses, the Moody diagram is often used to present the functional dependence of friction factor, f, on the Reynolds number, $\mathrm{Re}_D = \rho VD/\mu$, and the relative roughness, ε/D. However, the Moody diagram is unhandy for computer-based solutions, and a closed-form expression is desired. In the laminar regime, the usual expression is

$$f = \frac{64}{\mathrm{Re}_D} \qquad (3)$$

Several different representations are available for turbulent flow. In this paper the representation of Haaland (1983), Equation (4), is used.

$$f = \frac{0.3086}{\left[\log\left(\left(\frac{\varepsilon}{3.7D}\right)^{1.11} + \frac{6.9}{\mathrm{Re}_D}\right)\right]^2} \qquad (4)$$

Minor loss terms are sometimes expressed as equivalent lengths using the fully-rough friction factor, f_T, the asymptotic value of the friction factor for a given relative roughness. From the Haaland equation, the fully-rough friction factor becomes

$$f_T = \frac{0.3086}{\left[\log\left(\left(\frac{\varepsilon}{3.7D}\right)^{1.11}\right)\right]^2} \qquad (5)$$

With the aforementioned as the basis for piping system problem solution formulation, some examples of the unified approach will be examined and discussed.

EXAMPLES

Examples for series, parallel, and network piping systems will be explored in this section using the unified solution approach built around Equations (1) and (2) and the computational software system, Mathcad. Although Mathcad is the computational software system used in this paper, other computational software systems possess the same capability and could be used equally well.

Series Examples

Example 1 Problem Statement:

Water is to be pumped at the rate of 50 gpm from a lake to a storage tank. The free surface of the tank is 30 ft above the free surface of the lake. The pipe is 115 ft long, is constructed of schedule 80 pipe, and contains two 45-degree elbows and three 90-degree elbows. Find the increase in head of the pump and the power the pump delivers to the fluid.

Solution:
A schematic of the system is presented in Figure 2. The energy equation for the system becomes

$$\frac{P_A}{\gamma}+\frac{V_A^2}{2g}+Z_A=\frac{P_B}{\gamma}+\frac{V_B^2}{2g}+Z_B+f\,\frac{L}{D}\frac{V^2}{2g}+\frac{V^2}{2g}(K_{ent}+\sum K_{elbows}+K_{exit})-W_s\frac{g_c}{g} \quad (6)$$

The minor losses are the entrance, the elbows, and the exit. Since A and B are located at free surfaces open to the atmosphere, $P_A = P_B$ and $V_A = V_B = 0$. The energy equation thus reduces to the form

$$W_s = Z_B - Z_A + \frac{V^2}{2g}(f\,\frac{L}{D} + K_{ent} + 2K_{elbow-45} + 3K_{elbow-90} + K_{exit}) \quad (7a)$$

Since $Q = A \cdot V$ and $A = \frac{\pi D^2}{4}$ for a circular pipe, $\frac{V^2}{2g} = \frac{8 \cdot Q_2}{\pi^2 g D^4}$ and Equation (7a) can be expressed as

$$W_s = Z_B - Z_A + \frac{8 \cdot Q^2}{\pi^2 g D^4}(f\,\frac{L}{D} + K_{ent} + 2K_{elbow-45} + 3K_{elbow-90} + K_{exit}) \quad (7b)$$

The determination of the pump increase in head for a specified flow rate is a Category I series piping problem solution and can be solved directly. However, in this paper the generalized approach will be used. The Mathcad worksheet for the solution is provided in Figure 3. Most of the worksheet contains the specification of the system geometry, losses, and fluid properties. The definition of Reynolds number and functions for the friction factor and fully-rough friction factor are provided. The friction factor definition is piece-wise continuous with different expressions for the laminar and turbulent regimes. Transition is specified to occur at a Reynolds number of 2300. In Mathcad, the **Given** statement initiates a *Solve* block, and the **Find** statement specifies the unknown variable (or variables). In this example, the unknown is the increase in head of the pump, W_s. The solution is $W_s = 62.009\frac{ft \cdot lbf}{lbm}$. This problem is a straightforward example typical of those encountered in a first course in fluid mechanics. Consider a more complex version of this problem.

Example 2 Problem Statement:
If a pump imparts 2 hp to the fluid in the system of Example 1, what is the flow rate?

Solution:
This is more complex problem than Example 1. The reduced energy equation is essentially the same as Equation (7b) for Example 1. That portion of the Mathcad solution that differs from the solution of Example 1 (Figure 3) is given in Figure 4. The only difference is in the *Solve* block structure where two equations and two unknowns are specified. The additional equation is the definition of the power to the fluid. While Example 1 is simple and could be solved directly, Example 2 requires iteration. The flow rate that results in 2 hp being delivered to the fluid is 76.559 gal/min with a required pump increase in head of 103.346 ft-lbf/lbm. As confirmation of the accuracy of the solution, the power delivered to the fluid is computed from the solution results and is 2 hp as specified. This would be a challenging problem to work "by hand," but the unified approach is logical and straightforward. As in Example 1, the engineering aspect of the problem is in the formulation as a nonlinear system of two equations, but the solution via Mathcad is the same as for the simpler example and is transparent to the user.

Parallel Example

Example 3 Problem Statement
A parallel piping system, schematically illustrated in Figure 5, is to be analyzed. Table 1 presents characteristics of the two pipes in the parallel system.

Table 1. Pipe Characteristics for Parallel System

Pipe	L (ft)	D(in)	K	C	ε(ft)
1	3000	12	2	50	0.01
2	3000	8	1	100	0.0001

Oil with physical properties of 64.32 lbm/ft^3 and a viscosity of 0.00193 lbm/ft-sec is the fluid, and Z_A = 100 ft and Z_B = 80 ft. If a pump with an increase in head of 50 ft-lbf/lbm is placed in the system and if the pressure at A is to be the same as at B, find the total system flow rate and the flow rates in the individual pipes.

Solution:
The formulation of the system of equations for the solution invokes the behavior of a parallel system—namely, the flow rates add and the changes in head across each parallel line segment must be the same (uniqueness of pressure). Conservation of mass for this parallel system becomes

$$Q_T = Q_1 + Q_2 \tag{8}$$

And the head change from A to B for a parallel pipe segment can be expressed using the energy equation as

$$h_{A-B} = W_s \frac{g_c}{g} = Z_B - Z_A + \frac{8 \cdot Q^2}{\pi^2 g D^4}\left(f \frac{L}{D} + K + C \cdot f_T\right) \tag{9}$$

For a system composed of two parallel lines, three equations, one conservation of mass and an energy equation for each line, are required. For this system, the increase in head of the pump is given, so the unknowns are the flow rates, Q_T, Q_1, and Q_2.

The Mathcad worksheet for the solution of this parallel problem is given in Figure 6. The format is similar to Examples 1 and 2. Pipe parameters, fluid properties, and expressions for the Reynolds number and friction factors are provided, and the solution is obtained via a *Solve* block. The three equations discussed above are in the *Solve* block, and the results are obtained using the **Find** statement. The same principles were used in formulating the solution to this parallel problem as were used to formulate the solutions of the series problems. Once the parallel problem was formulated, the *Solve* block and the **Find** statement were used to obtain the solution.

Network Example

Example 4 Problem Statement
A piping network composed of seven lines and two loops is illustrated in Figure 7(a). Characteristics of the pipes are provided in Table 2, and the fluid is water. Find the flow rate in each line of the piping network.

Table 2. Pipe Characteristics for Network System

Pipe	L (ft)	D(in)	K	C	ε(ft)
1	2000	12	0	0	0.00015
2	2000	8	0	0	0.00015
3	3000	6	0	0	0.00015
4	4000	6	0	0	0.00015
5	1000	8	0	0	0.00015
6	3000	8	0	0	0.00015
7	2000	8	0	0	0.00015

Solution:
Network problems, such as this one, can be solved by the Hardy-Cross procedure (Hodge and Taylor, 1999). However, the Hardy-Cross procedure, which is formulated using the aforementioned principles, introduces a loop-correction factor that is used in a Newton-Raphson iterative procedure. An alternative to the Hardy-Cross procedure is to implement directly the three principles delineated in the Background segment of this paper.

As illustrated in Figure 7(a), the network is composed of six nodes, seven pipes, and two loops. Conservation of mass must be enforced at each node, and the energy equation must hold for each pipe. Uniqueness of pressure requires that the sum of the changes in pressure (or head) around each loop be zero. For this example, the unknowns are the seven flow rates ($Q_1 \ldots Q_7$). The system of equations required for the solution to this network must, therefore, contain seven equations. The seven equations are five conservation of mass expressions and two uniqueness of pressure statements. Conservation of mass expressions can be written for all six nodes, but only five of the expressions will be independent. Initial guesses on all flow rates are needed for the *Solve* block. The initial guesses do not have to satisfy conservation of mass at each node, but the ones used in this example do. The conservation of mass statement at each node must have the total inflow equal to the total outflow at a node. Suitable conservation of mass expressions for the initial flow rate directions are as follows:

$$\begin{aligned} 3 &= Q_1 + Q_4 + Q_5 \\ Q_1 + Q_2 &= Q_7 \\ 2 &= Q_6 + Q_7 \\ 1 + Q_2 &= Q_3 \\ Q_5 &= Q_6 \end{aligned} \qquad (10)$$

The major and minor head losses in a pipe segment can be expressed in functional form as

$$h(Q,K,C,L,D) = \frac{8 \cdot Q|Q|}{\pi^2 g D^4}\left(f\frac{L}{D} + K + C \cdot f_T \right) \qquad (11)$$

where $Q|Q|$ carries the sign convention (a positive flow rate yields a positive head loss). For loop 2, the sum of the head changes around the loop must equal zero or

$$h(Q_5,K_5,C_5,L_5,D_5) + h(Q_6,K_6,C_6,L_6,D_6) - h(Q_7,K_7,C_7,L_7,D_7) - h(Q_1,K_1,C_1,L_1,D_1) = 0 \qquad (12)$$

Equations (10) and (12) plus the analogous equation for loop 1 constitute the system of equations needed to describe the system. Figure 8 presents the Mathcad work sheet implementing the solution to this network problem. The pipe characteristics are defined and the Reynolds number and the friction factors expressions are presented. The minor loss coefficients vectors, K and C, are indicated. But as with the series and the parallel systems, the problem solution is accomplished by the *Solve* block—in this example the *Solve* block contains seven equations. The solution is provided by the **Find** statement and in given in Figure 8 and illustrated in Figure 7c. The flow rate is line 2 was guessed as being clockwise in loop 1, but the solution shows the flow rate in line 2 to be counterclockwise in loop 1. The use of $Q|Q|$ in the energy equation expressions provides the capability of the system of equation to handle flow rates initially guessed to be in the wrong direction. The *Solve* block in this network problem contains seven equations and is, thus, more complicated than the *Solve* block of the previous examples, but the same principles were used in formulating the equations in the *Solve* block.

PEDAGOGICAL INFERENCES

The purpose of this paper is to discuss a unified method of solving piping problems. In all the examples explored in this paper, the same three principles were used in formulating an equation or a system of equations for the solution. In most undergraduate courses, the treatments of series, parallel, and network problems are distinct and emphasize the arithmetic sequence required to solve the equation or equations formulated as the problem solution. In this paper attention has been directed to formulating the solutions to series, parallel, and network problems, but the arithmetic has been accomplished by using the *Solve-Find* structure of Mathcad. Other computational software systems (Mathematics, Matlab,….) offer the same capability, albeit in different formats, but with the same results.

Anecdotally, students appreciate the attention to problem formulation using the three principles built around statements of conservation of mass and energy and uniqueness of pressure at a point. The use of Mathcad with its *Solve-Find* structure relieves the student from assimilating different numerical techniques ("procedures") to solve a non-linear equation or a system of non-linear equations. The net result is that more involved and more realistic problems can be assigned. With less time spent on arithmetic, more time is available for students to engage is higher-level synthesis and understanding.

CONCLUSIONS

Examples illustrating a unified approach to solutions of series, parallel, and network piping problems have been presented and discussed. Pedagogical aspects of using a unified approach for the solution of piping problems are examined. The unified approach offers advantages in providing students with capability to solve more “real world” problems and to engage in higher order activities.

REFERENCES

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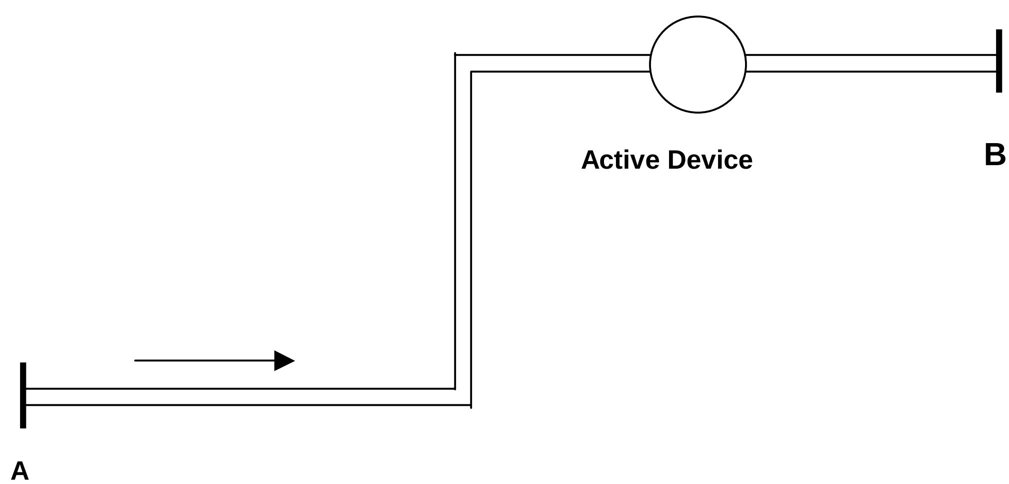

Figure 1. Pipe Segment Schematic

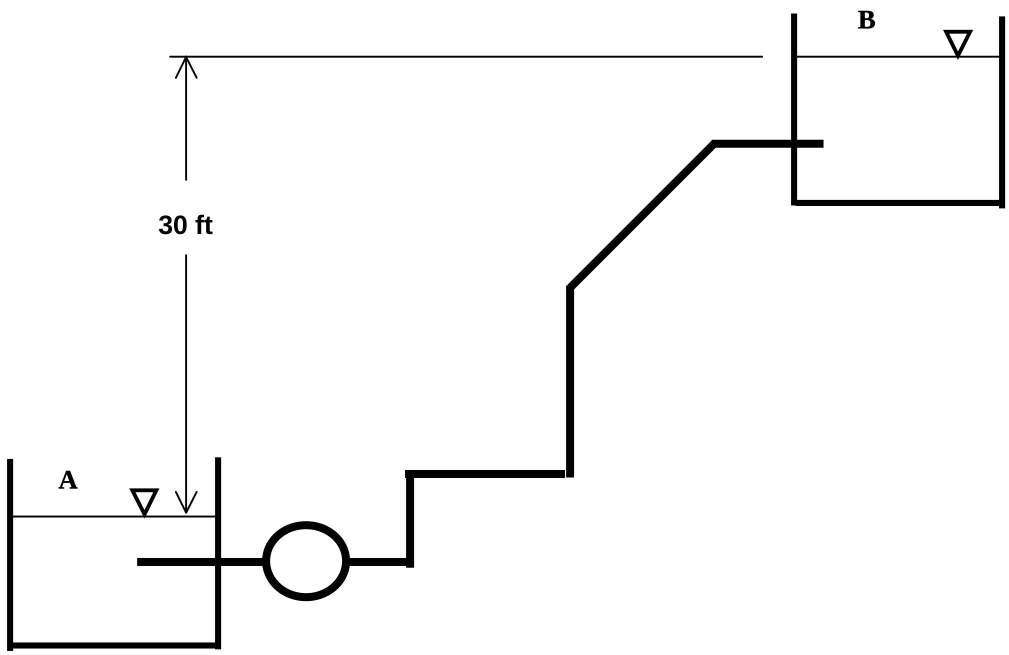

Figure 2. Example 1 Schematic

ORIGIN≡ 1 Set origin for counters to 1 from the default value of 0.

Input the pipe geometry:

Diameter in inches Length in feet Roughness in feet:

$D := \begin{pmatrix} 1.5 \\ 1.5 \end{pmatrix} \cdot in$ $L := \begin{pmatrix} 15 \\ 100 \end{pmatrix} \cdot ft$ $\varepsilon := \begin{pmatrix} 0.00015 \\ 0.00015 \end{pmatrix} \cdot ft$

Input the system boundary (initial and end) conditions, the loss coefficients, and the fluid propeties:

Pressures in psi Elevations in feet:

$\begin{pmatrix} P_a \\ P_b \end{pmatrix} := \begin{pmatrix} 0 \\ 0 \end{pmatrix} \cdot \frac{lbf}{in^2}$ $\begin{pmatrix} Z_a \\ Z_b \end{pmatrix} := \begin{pmatrix} 0 \\ 30 \end{pmatrix} \cdot ft$

K factor Equivalent length Number of pipes

$K := \begin{pmatrix} 0.78 \\ 1 \end{pmatrix}$ $C := \begin{pmatrix} 32 \\ 90 \end{pmatrix}$ $N := \text{length}(D)$

Density in lbm/ft^3 Viscosity in lbm/ft-s

$\rho := 62.4 \frac{lb}{ft^3}$ $\mu := 0.000658 \frac{lb}{ft \cdot sec}$

Input the flow rate in cfs: $Q := 50 \cdot \frac{gal}{min}$

Define constants and adjust units for consistency: $g := 32.174 \frac{ft}{sec^2}$ $g_c := 32.174 \frac{ft \cdot lb}{lbf \cdot sec^2}$

Define the functions for Reynolds number, fully-rough friction factor, and friction factor:

$Re(q,d) := \frac{4 \cdot \rho \cdot q}{\pi \cdot d \cdot \mu}$ $f_T(d,\varepsilon) := \frac{0.3086}{\left[\log\left[\left(\frac{\varepsilon}{3.7 \cdot d}\right)^{1.11}\right]\right]^2}$

$$f(q,d,\varepsilon) := \begin{cases} \dfrac{0.3086}{\left[\log\left[\dfrac{6.9}{Re(q,d)} + \left(\dfrac{\varepsilon}{3.7 \cdot d}\right)^{1.11}\right]\right]^2} & \text{if } Re(q,d) > 2300 \\ \dfrac{64}{Re(q,d)} & \text{otherwise} \end{cases}$$

$W_s := 100 \cdot ft \cdot \frac{lbf}{lb}$ (Initial guess of pump increase in head.)

Given

$$W_s \cdot \frac{g_c}{g} = \frac{P_b - P_a}{\rho \cdot g} \cdot g_c + Z_b - Z_a + \sum_{i=1}^{N} \frac{8}{\pi^2} \cdot \frac{Q^2}{g \cdot (D_i)^4} \cdot \left(f(Q, D_i, \varepsilon_i) \cdot \frac{L_i}{D_i} + K_i + C_i \cdot f_T(D_i, \varepsilon_i) \right)$$

$W_s := \text{Find}(W_s)$ $W_s = 62.009 ft \cdot \frac{lbf}{lb}$

Pump power (input to fluid): $Power := Q \cdot \rho \cdot W_s$ $Power = 0.784 hp$

Figure 3. Mathcad Worksheet for Example 1

$W_s := 10 \cdot ft \cdot \frac{lbf}{lb}$ $\quad Q := 50 \cdot \frac{gal}{min}$ (Initial guesses of pump increase in head and flowrate.)

Given

$2 \cdot hp = \rho \cdot Q \cdot W_s$

$$W_s \cdot \frac{g_c}{g} = \frac{P_b - P_a}{\rho \cdot g} \cdot g_c + Z_b - Z_a + \sum_{i=1}^{N} \frac{8}{\pi^2} \cdot \frac{Q^2}{g \cdot (D_i)^4} \cdot \left(f(Q, D_i, \varepsilon_i) \cdot \frac{L_i}{D_i} + K_i + C_i \cdot f_T(D_i, \varepsilon_i) \right)$$

$\begin{pmatrix} W_s \\ Q \end{pmatrix} := Find(W_s, Q)$ $\quad W_s = 103.346 \frac{ft \cdot lbf}{lb}$ $\quad Q = 76.559 \frac{gal}{min}$

Pump power (input to fluid): $Power := Q \cdot \rho \cdot W_s$ $\quad Power = 2\, hp$

Figure 4. Mathcad Solve Block for Example 2

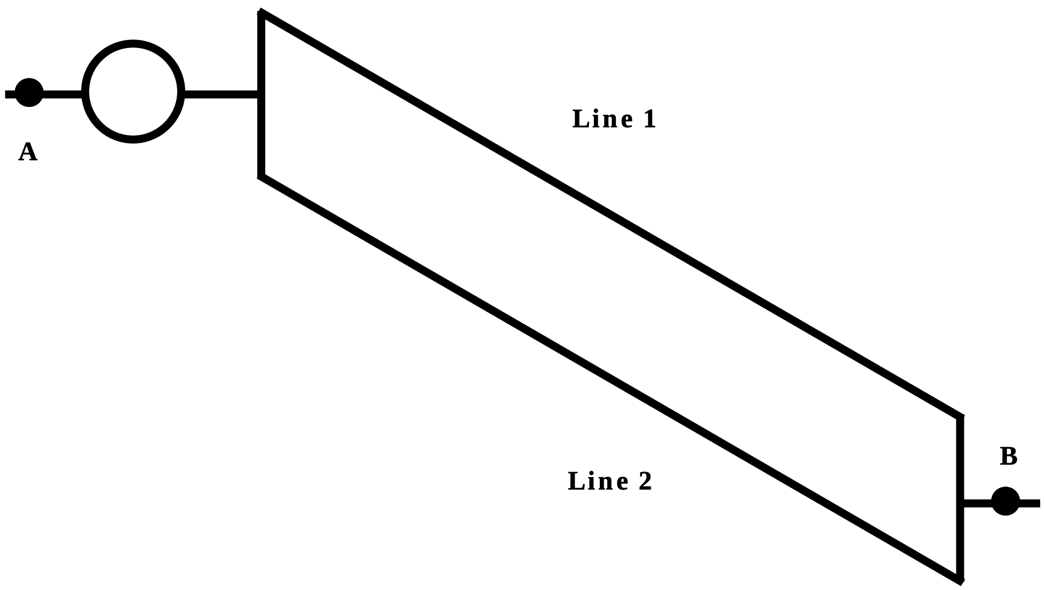

Figure 5. Parallel System Schematic

$ORIGIN \equiv 1$ Set origin for counters to1 from default value of 0.

Input the pipe geometry and the elevation difference, the loss coefficients, and the physical properties:

$$D := \begin{pmatrix} 12 \\ 8 \end{pmatrix} \cdot in \quad L := \begin{pmatrix} 3000 \\ 3000 \end{pmatrix} \cdot ft \quad \varepsilon := \begin{pmatrix} 0.01 \\ 0.0001 \end{pmatrix} \cdot ft \quad \begin{pmatrix} Z_a \\ Z_b \end{pmatrix} := \begin{pmatrix} 100 \\ 80 \end{pmatrix} \cdot ft$$

K factor Equivalent length Number of pipes

$$K := \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad C := \begin{pmatrix} 50 \\ 100 \end{pmatrix} \quad N := \text{length}(D)$$

Density in lbm/ft^3 Viscosity in lbm/ft-s

$$\rho := 64.35 \cdot \frac{lb}{ft^3} \quad \mu := 0.00193 \frac{lb}{ft \cdot sec}$$

Define constants and adjust units for consistency: $g := 32.174 \frac{ft}{sec^2}$ $g_c := 32.174 \frac{ft \cdot lb}{lbf \cdot sec^2}$

Define the functions for Reynolds number and the friction factors:

$$Re(q, D) := \frac{4 \cdot \rho \cdot q}{\pi \cdot D \cdot \mu} \quad f_T(D, \varepsilon) := \frac{0.3086}{\left[\log\left[\left(\frac{\varepsilon}{3.7 \cdot D}\right)^{1.11}\right]\right]^2}$$

$$f(q, D, \varepsilon) := \begin{cases} \dfrac{0.3086}{\left[\log\left[\dfrac{6.9}{Re(q,D)} + \left(\dfrac{\varepsilon}{3.7 \cdot D}\right)^{1.11}\right]\right]^2} & \text{if } Re(q, D) > 2300 \\ \dfrac{64}{Re(q,D)} & \text{otherwise} \end{cases}$$

Setup Solve Block by defining specified inputs and guessed values:

$$Q_T := 5.0 \cdot \frac{ft^3}{sec} \quad Q1 := \frac{Q_T}{N} \quad Q2 := \frac{Q_T}{N} \quad W_s := 50 \cdot ft \cdot \frac{lbf}{lb}$$

Given

$$Q_T = Q1 + Q2$$

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q1)^2}{g \cdot (D_1)^4} \cdot \left(f(Q1, D_1, \varepsilon_1) \cdot \frac{L_1}{D_1} + K_1 + C_1 \cdot f_T(D_1, \varepsilon_1) \right)$$

$$W_s \cdot \frac{g_c}{g} = Z_b - Z_a + \frac{8}{\pi^2} \cdot \frac{(Q2)^2}{g \cdot (D_2)^4} \cdot \left(f(Q2, D_2, \varepsilon_2) \cdot \frac{L_2}{D_2} + K_2 + C_2 \cdot f_T(D_2, \varepsilon_2) \right)$$

$$\begin{pmatrix} Q_T \\ Q1 \\ Q2 \end{pmatrix} := \text{Find}(Q_T, Q1, Q2)$$

$$Q_T = 7.481 ft^3 sec^{-1} \quad Q1 = 4.839 ft^3 sec^{-1} \quad Q2 = 2.642 ft^3 sec^{-1}$$

Figure 6. Mathcad Worksheet for Parallel System

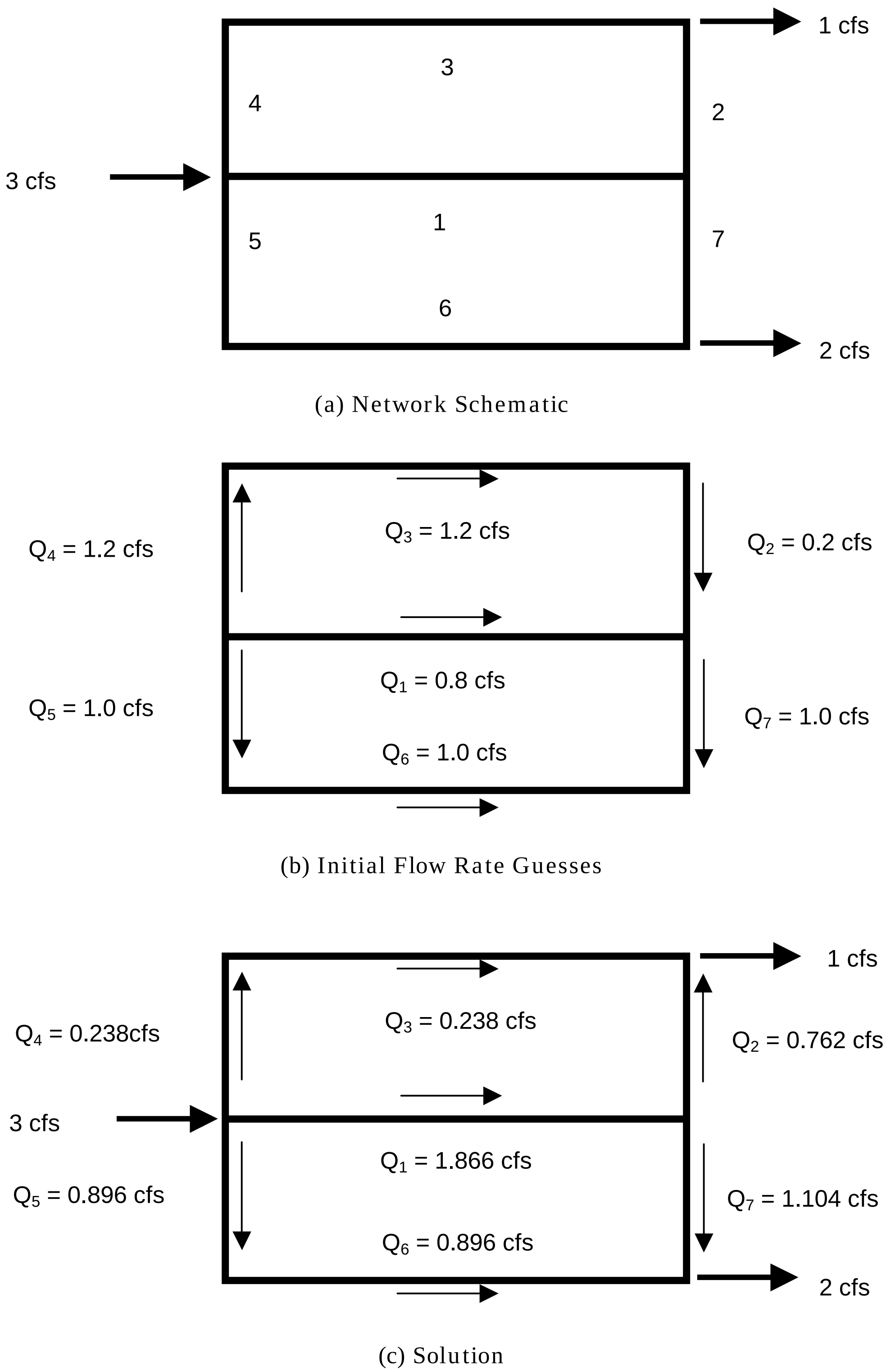

Figure 7. Network Problem Schematic and Solution

ORIGIN≡ 1 Reset counter to start at 1 rather the default value of 0.

Input the pipe geometry

$$L := \begin{pmatrix} 2000 \\ 2000 \\ 3000 \\ 4000 \\ 1000 \\ 3000 \\ 2000 \end{pmatrix} \cdot ft \qquad Q := \begin{pmatrix} 0.8 \\ 0.2 \\ 1.2 \\ 1.2 \\ 1.0 \\ 1.0 \\ 1.0 \end{pmatrix} \cdot \frac{ft^3}{sec} \qquad \varepsilon := \begin{pmatrix} 0.00015 \\ 0.00015 \\ 0.00015 \\ 0.00015 \\ 0.00015 \\ 0.00015 \\ 0.00015 \end{pmatrix} \cdot ft \qquad D := \begin{pmatrix} 12 \\ 8 \\ 6 \\ 6 \\ 8 \\ 8 \\ 8 \end{pmatrix} \cdot in$$

Define constants and unit adjustments: $g := 32.174 \cdot \frac{ft}{sec^2}$

Define physical properties: $\nu := 0.000016 \cdot \frac{ft^2}{sec}$

The usual functions for friction factor must be defined:

$$Re(q,d) := \frac{4 \cdot |q|}{\pi \cdot d \cdot \nu} \qquad f_T(d,\varepsilon) := \frac{0.3086}{\left[\log\left[\left(\frac{\varepsilon}{3.7 \cdot d}\right)^{1.11}\right]\right]^2}$$

$$f(q,d,\varepsilon) := \begin{cases} \begin{cases} \dfrac{0.3086}{\left[\log\left[\dfrac{6.9}{Re(q,d)} + \left(\dfrac{\varepsilon}{3.7 \cdot d}\right)^{1.11}\right]\right]^2} & \text{if } Re(q,d) > 2300 \\ \dfrac{64}{Re(q,d)} & \text{otherwise} \end{cases} & \text{if } |q| > 0 \\ 1 & \text{otherwise} \end{cases}$$

Define the minor loss coefficients K and the equivalent-lengths C:

$$K := (0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)^T \qquad C := (0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0)^T$$

Define the loss function for each line using the friction factor major loss expression:

$$h(Q,K,C,L,D) := \frac{8 \cdot Q \cdot |Q|}{\pi^2 \cdot g \cdot D^4} \cdot \left(f(Q,D,\varepsilon) \cdot \frac{L}{D} + K + C \cdot f_T(D,\varepsilon)\right)$$

Figure 8. Mathcad Worksheet for Pipe Network Solution

Given

$$3 \cdot \frac{ft^3}{sec} = Q_1 + Q_4 + Q_5$$

$$Q_1 + Q_2 = Q_7$$

$$2 \cdot \frac{ft^3}{sec} = Q_6 + Q_7$$

$$1 \cdot \frac{ft^3}{sec} + Q_2 = Q_3$$

$$Q_5 = Q_6$$

$$h(Q_4, K_4, C_4, L_4, D_4) + h(Q_3, K_3, C_3, L_3, D_3) + h(Q_2, K_2, C_2, L_2, D_2) - h(Q_1, K_1, C_1, L_1, D_1) = 0$$

$$h(Q_5, K_5, C_5, L_5, D_5) + h(Q_6, K_6, C_6, L_6, D_6) - h(Q_7, K_7, C_7, L_7, D_7) - h(Q_1, K_1, C_1, L_1, D_1) = 0$$

$$Q := \mathrm{Find}(Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7)$$

$$Q = \begin{pmatrix} 1.866 \\ -0.762 \\ 0.238 \\ 0.238 \\ 0.896 \\ 0.896 \\ 1.104 \end{pmatrix} \frac{ft^3}{sec}$$

Figure 8. Mathcad Worksheet for Pipe Network Solution (Concluded)