Utilizing Mathematics Software to Support Mathematics/Science Connections to the Automotive Industry for Secondary Mathematics

Susan Beseler[1], Mike Bestor[2], Kyle Manley[3], Julia Upton[4]

Abstract – The National Science Foundation's Graduate Teaching Fellows in K-12 Education (GK-12) Program at the University of Alabama [1], supports teaching fellowships for graduate students in the sciences, technology, engineering, and mathematics (STEM) disciplines. The NSF graduate fellows participate in interdisciplinary teams to develop materials to be utilized in secondary science and mathematics classrooms. The project's goal is to enhance the educational practices in use by elementary and secondary teachers and students with emphasis on materials that improve critical thinking skills, effectively reaching students in every grade, academic performance level, cultural background and learning style. An important task of each graduate fellow will be the development of new learning modules for novice learners, created in the process of development and implementation of this program. The specific context for the UA GK-12 activities is Alabama's growing automotive industry and, in accordance, the following provides various modules for use in secondary mathematics classrooms to connect algebra, trigonometry and calculus concepts to the automotive industry through instructional technology.

Keywords: GK-12, trigonometry, calculus, automotive industry, secondary

EXPLORING DISTANCE AND VELOCITY AS FUNCTIONS OF TIME

The learning module utilizes interactive dynamic geometry software, The Geometer's Sketchpad®. Through this interactive instructional technology tool, the students are able to graph functions, plot points, perform calculations and create geometric constructions all in a user-friendly software forum. With minimal technical proficiency, users navigate drop-down menus to create the mathematical models necessary for exploration. This particular activity introduces the concept of an antiderivative interpreted as an area under a function's curve. First, students calculate areas under graphs of linear functions by using familiar formulas for areas of polygons. Following the initial calculations, they are encouraged to design their own methods of evaluating an area under a graph of a non-linear function. After several iterations to achieve better approximations of this area, a single formula that more efficiently and accurately accomplishes the task is introduced. Students are asked to determine the relationship between this function and the formula for the original nonlinear function. This activity is most appropriate for students who have recently completed the course portion on derivatives and are ready to be introduced to integration. The activity also reinforces understanding of functions and their domains.

Problem Statement:

A Short Trip.

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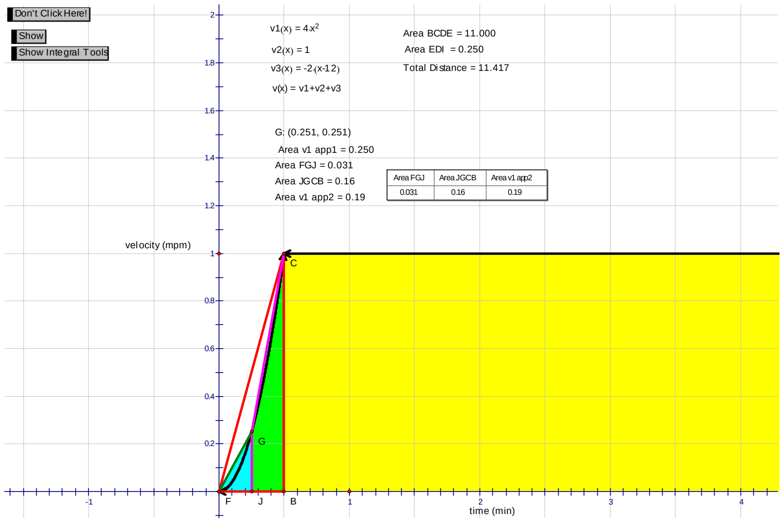
[1] University of Alabama, Secondary CTL, Tuscaloosa, besel001@bama.ua.edu

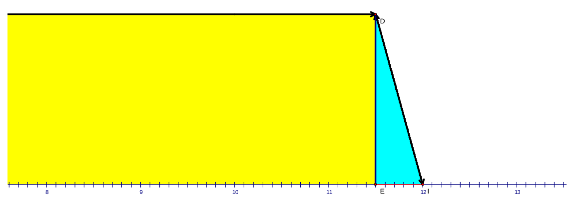
[2] University of Alabama, Chemical Engineering, Tuscaloosa, besto001@bama.ua.edu

[3] University of Alabama, Mathematics Dept., Tuscaloosa, manle007@bama.ua.edu

[4] University of Alabama, Mathematics Dept., Tuscaloosa, upton002@bama.ua.edu

You get into your car and accelerate from 0 to 60 miles per hour in 30 seconds (assume your velocity for this portion of the trip is given by the function v1(x) = $4x^2$ where velocity is in miles per minute and x stands for time in minutes). You maintain a constant speed of 60 miles per hour for 11 minutes and then decelerate to a complete stop in 30 seconds (deceleration portion is given by $v_3(x) = -2(x-12)$)





$v1(x) = 4 \cdot x^2$

$v2(x) = 1$

$v3(x) = -2 \cdot (x-12)$

$v(x) = v1+v2+v3$

Area BCDE = 11.000

Area EDI = 0.250

Total Distance = 11.417

G: (0.251, 0.251)

Area v1 app1 = 0.250

Area FGJ = 0.031

Area JGCB = 0.16

Area v1 app2 = 0.19

Area FGJ	Area JGCB	Area v1 app2
0.031	0.16	0.19

Guiding Questions:

What is the total distance you traveled?

1. Can the answer be determined from your speedometer's reading? Why?

2. Open the sketch "Distance Original" and plot the functions $v_1(x)$, $v_2(x)$, $v_3(x)$, where v_1, v_2, v_3 refer to acceleration, constant speed, and deceleration portions of your trip, respectively. Plot only the parts of the functions relevant for the chosen domains (for example, $v_1(x)$ is only to be plotted for $0<x<0.5$ minutes, etc.).

3. Type into the sketch the formula for v(*x*), which describes the total velocity function for your trip.

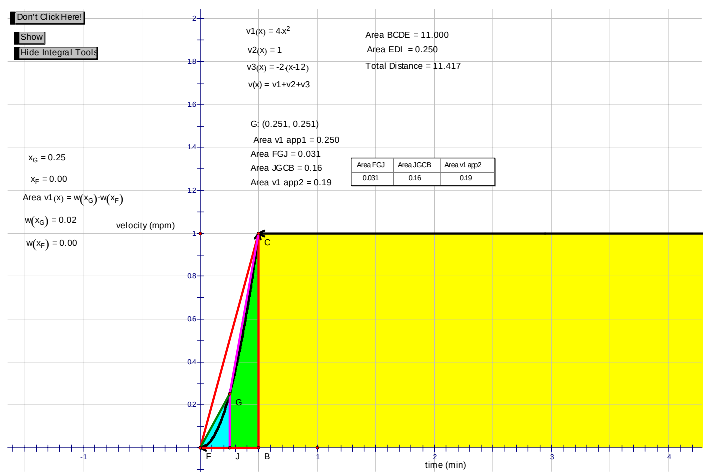
4. How is the distance calculated from a constant speed and travel time? To what portion of your trip can this formula be applied? What does this formula represent geometrically (examine the graph)? Use the software calculator to determine the distance for this part of the trip. Type the answer in your sketch in a complete sentence, stating what the number refers to and its units.

5. What polygon area formula can be used to determine distance traveled during the deceleration part of the trip? Use the software calculator to calculate this distance. Type the answer in your sketch in a complete sentence, stating what the number refers to and its units.

6. How can the distance for the acceleration part of the trip be approximated? Use the software calculator to calculate this distance. Type the answer in your sketch in a complete sentence, stating what the number refers to and its units. Is your approximation larger or smaller than the actual distance? How can your approximation be improved?

7. Use polygon area formulas to obtain two more approximations of the distance for the acceleration part of the trip.

8. There's a simple formula that allows you to calculate the area under a graph of a non-linear function. Click on "Don't Click Here" button to obtain the result of the calculation for the area under v_1 using this formula, then on "Show" to reveal this formula. How was the result obtained from the formula (i.e. what value(s) for *x* were used)?



9. What's the relationship between the function v1 and the formula revealed in #8? (Hint: recall the Power Rule of differentiation)

10. Calculate the total distance for your trip. Type the answer in your sketch in a complete sentence, stating what the number refers to and its units. Save all your work!

Upon completion of the activity, follow-up discussion and further applications are explored. The use of concept-building as a precursor to procedural formulas creates a learning environment following the constructivist tradition in which students explore and construct their own meaning, hereby, creating foundation of meaning that most optimizes the learning outcomes.

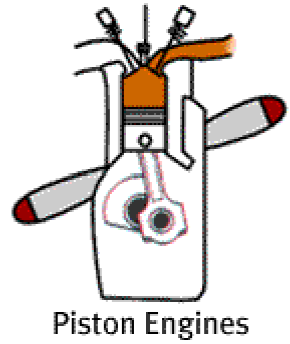
EXPLORING ENGINE DESIGN AND THE LAW OF SINES

This example provides a visual demonstration of how an engine works and how to utilize basic geometric shapes to reconstruct the design and analyze various mathematical relationships, namely the Law of Sines. Engine design and operation can be explained through this model to demonstrate simple models and how important simulation is to their respective field. At all levels, the use of modeling to provide visual exploration tools into relationships can be valuable in aiding the process of mathematical concept building.

Similar to the university-level computer aided design (CAD) systems, dynamic geometry software such as Geometer's Sketchpad® allows for manipulation and animation in visual models. A distinct advantage of dynamic geometry software exists in its ease of use in relative user-friendly programming capabilities. As outlined by the National Council of Teachers of Mathematics [2], the use of real-life connections throughout elementary, secondary and post-secondary mathematics courses is fundamental in creating the optimal learning environment. The use of dynamic geometry software in this capacity is provided in the following activity.

Students are provided with a completed sketch that is used in conjunction with a set of guiding questions that provides a structure for exploration into the sine relationships present within a piston engine model.

Figure 1



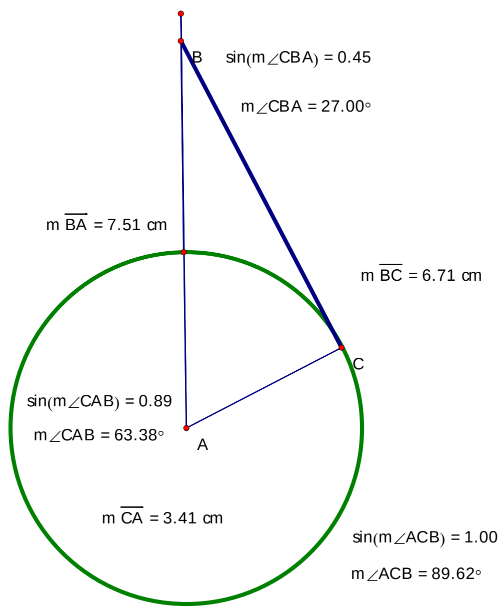
Guiding Questions:

•In the animation (http://www.howstuffworks.com/engine4.htm) what type of path does the piston follow? (linear, circular, , parabolic, elliptical or some other type) (Figure 1)

•In the animation, what type of path does the crankshaft follow?

•Sketch a simplified version of this engine type. You can only use a circle and three segments.

•What "MATH" things are in the picture? Name four objects that you could classify, measure, or calculate in mathematical terms.



•Law of ________________ (fill in the blank when you are finished)

•What, if any, patterns do you see? Is there a relationship between the "pairs"?

•The sine measurements and the opposite lengths. What do you notice?

•Watch the ratios! What do you see now?

•Given the following information use the Law of Sines to solve the triangle.

a=10, C=17°, c=11

A=40°, B=60°, c=20

Now go back to the engine pictures and compare to the sketchpad model.

$\frac{\sin(m\angle CAB)}{m\overline{BC}} = 0.13\ \text{cm}^{-1}$ $\frac{\sin(m\angle CBA)}{m\overline{CA}} = 0.13\ \text{cm}^{-1}$ $\frac{\sin(m\angle ACB)}{m\overline{BA}} = 0.13\ \text{cm}^{-1}$

#5 SinA / a #6 SinB / b #7 SinC / c

#8 Hide measurements #9 Watch the Ratios! What do you see?

•Name 3 things that are different, the same, missing or added in the model.

By utilizing dynamic geometry software for exploring trigonometric relationships, dynamic models can be efficiently produced for secondary mathematics curricula. In this module the synchronized motion of crankshaft and piston provide a forum for examining a trigonometric relationship namely, the Law of Sines.

This particular learning module was introduced to a first semester Trigonometry class in a large urban high school setting. With minimal previous exposure to this type of exploration activity and open-ended questioning, many of the students were weak in written expression forms of mathematical communication and required guidance during the writing portion of the module. Overall, however, the students reported that they enjoyed the opportunity to experience "real life" applications of mathematical concepts and were quick to become familiar with the software technicalities. Formal procedural assessments on the Law of Sines, also indicated that student achievement was improved from previous lessons.

Overall, both of the learning modules provided students with the opportunity to experience mathematical learning in a manner that not only provides real-life connections but is formatted to optimize the powerful, dynamic visualization tools currently available. By utilizing open-ended questioning techniques and dynamic interactions with technological tools, namely dynamic geometry software, the investigative nature of these modules supports the cognitive model building process. The common success of these classroom applications in utilizing dynamic geometry software in constructivist pedagogy supports the need for further exploration into these types of learning modules.

REFERENCES

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Susan Beseler

Ph.D. candidate, Secondary Curriculum, Teaching and Learning/Secondary Mathematics Education, University of Alabama. Research interests: technology in mathematics, pedagogical content knowledge and teacher preparation programs in mathematics. Current assignment: University of Alabama, Graduate Research and Teaching Assistant; responsibilities include instructing the course "Technology for Mathematics Teacher," and supervising secondary mathematics practicum students in field-based experiences. Previous experience: Winona State University, Winona, Minnesota, Mathematics Instructor. Experience also includes secondary and middle school mathematics along with numerous conference presentations.

Michael Allen Bestor

B.S. Chemical Engineering from Mississippi State (May 2004), Currently pursuing a M.S. Chemical Engineering from University of Alabama (May 2004-Present), Previous research: Environmental Effects of Heavy Metals Used in Small Arms Training, Utilizing Phosphate Binders to Immobilize Heavy Metals Used in Small Arms Training

(funded by the US Army Environmental Center). Research Interest: Improving Catalysts for Hydrogen Fuel Production and Storage for Proton Exchange Membrane Fuel Cells (funded by US Department of Energy)

James Kyle Manley
B.S., Mathematics, Mississippi College, May 2004
Currently Working Toward a M.A. in Applied Mathematics at University of Alabama
NSF GK-12 Graduate Fellow currently working at Bryant High School with Algebra II and Trigonometry courses.

Julia Tumasova Upton

B.A. (Philosophy), B.S. (Chemical Engineering), Prior experience includes process engineer and a Research Scientist for Kimberly-Clark Co. (1998-2003), currently working on Ph.D. in Pure Mathematics, Research interest - Galois Theory.