Arches: A Neglected Topic in Structural Analysis Courses

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Abstract

This paper reports on an experience carried out by the author to include the topic of Arches in a course on Structural Analysis II. The paper shows that the most popular textbooks do not address the topic, by considering the number of pages dedicated to arches. A strategy was implemented in which the instructor motivated the students to investigate about arches, with emphasis on their structural importance, their historical importance, their present day importance, and their esthetic importance. The students carried out projects in which the arches were analyzed by means of frame elements. The five structural themes were the numerical modeling of arches using frame elements, analysis of Romanesque and Gothic cathedrals, arch-supported bridge, and arch-suspended bridge. The evaluation of the experience was based on a questionnaire and showed highly positive results.

Introduction

Arch structures have a double feature in terms of civil engineering education: On the one hand, they have been used throughout the history of architecture as a main structural component. This is also true of present day construction, in which arches play a major role in the design of bridges, some building types, and other structures. On the other hand, they are neglected by the civil engineering curricula throughout the nation. Arch structures could (and should) be taught in Structural Analysis, a course that is obligatory in all civil engineering programs. In some schools there are two Structural Analysis courses (at the University of Puerto Rico they are INCI 4021 and INCI 4022, with a total of 6 credits), while other schools teach one course with 3 or 4 credits.

First, this paper contains a review of 20 books on structural analysis, in order to show that an average of only 2% of the texts is dedicated to arches. Second, the paper illustrates the arguments used by this instructor to highlight the historic and contemporary importance of arches in real structures. Having established the importance of this neglected topic, the paper reports on strategies to insert the analysis of arch structures by modeling them using frame elements.

Arches in Structural Analysis Books

The first stage in this work was the identification of what topics about arches were covered in textbooks on structural analysis, at least those more commonly used in civil engineering courses in the USA. A list of books is given in the appendix, including the identification of the book (title, authors, publishing house and year of publication), the coverage of arches (pages dedicated to arches, total number of pages, the ratio of arch pages to total number), and some comments about what special contents were covered (for example, only isostatic problems, only a definition of arch is given, the cover page of the book has an arch, etc.). The total number of pages in each book excludes Appendix, Bibliography, Index. The pages dedicated to arches include pages in which arches are mentioned.

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A total of 21 books were examined. In 8 books there was no mention of arches, and in 4 other cases the number of pages was less than 1% of the total number in the book. In 6 cases the pages of arches were between 1 and 2%. In 3 cases the book dedicated between 2 and 5% to arches, although mainly to isostatic arches. The main conclusion is that arches are not covered in any detail in structural analysis books, for reasons that are not explained in the books.

On the Motivation to Study Arches

The instructional strategy followed by the author in the course Structural Analysis II in 2003 was divided in two stages. First, the instructor wrote a document with a motivation to study the topic and presented it in class, but without specific reference to structural modeling. Second, the students used a frame analysis software (SAP 2000) to understand ways of modeling arches. They learned the use of the program in previous classes of the course. The textbook used in this course is [Hibbeler, 5] which only dedicates 1.4% to arches.

The motivating document (and its presentation in class) presented the following arguments:

(a) Argument of structural behavior: Because of the curvature, arches combine the advantages of bending (which is typical of frame elements), plus the advantages of compression (as in columns), and that the two mechanisms contribute to resist external loads normal to the middle line of the arch. Furthermore, arches can resist normal forces by developing only compression, so that they are adequate to build with materials that cannot withstand tension.

(b) Argument of presence in nature: There are many examples of natural arches, and many of them are collected in a website [The Natural Bridge and Arch Society, 6]. Arches are one of the most typical formations in places with erosion.

(c) Argument of historical importance: Arches played a dominant role as construction elements in the Roman Empire (about I-II Century). Examples of arches are found in aqueducts, buildings, bridges over narrow rivers. The Arab civilization included arches in building and passed this technique to the Medieval European construction [Binding, 2; Henry-Claude, M. et al., 3]. The Medieval world was full of arches in various functions, in Cathedrals, Castles, civic and religious buildings, and bridges. Both Romanesque and Gothic cathedrals used arches as main supporting elements.

(d) Argument of present day importance: Arches are used in concrete as well as in steel bridges, in arch dams, and in industrial, exhibition and commercial buildings [National Geographical Society, 5]. The concept of a deck supported by an arch from above via vertical tensors, which is so popular in bridges, has also been incorporated in the construction of buildings to leave the base floor free from columns, as in railways stations.

(e) Argument of esthetic importance: Curved elements in general and arches in particular are designs with esthetic value. Many architects use arches as sculptures in open spaces or as symbolic elements.

Projects Solved by the Students

The class (18 students) was divided in groups to solve five individual projects related to arches. The five structural themes were the numerical modeling of arches using frame elements, analysis of Romanesque and Gothic cathedrals, arch-supported bridge, and arch-suspended bridge. Each project lasted for three weeks, in which the students modeled the problem using SAP 2000 [Anonymous, 1]. In previous assignments, the students had experience with the analysis of trusses, beams and frames using SAP 2000, but this was the first time that they had to model a problem by themselves. A brief summary of the main achievements of some groups is given next.

The first group investigated the number of straight elements necessary to represent a circular arch. They considered self weight plus a concentrated load at the apex, and studied discretizations of half the arch using symmetry and 1, 2, 4, 8 and 16 elements along the semi-arch. This was the first time they studied convergence of a model, and the results are plotted in Figure 1. They concluded that using 4 elements gives a very good approximation for the displacements and for the moments, but for the shear it may be necessary to use 8 elements. This produced a good discussion in the group, and they learned that convergence depends on the specific variable considered.

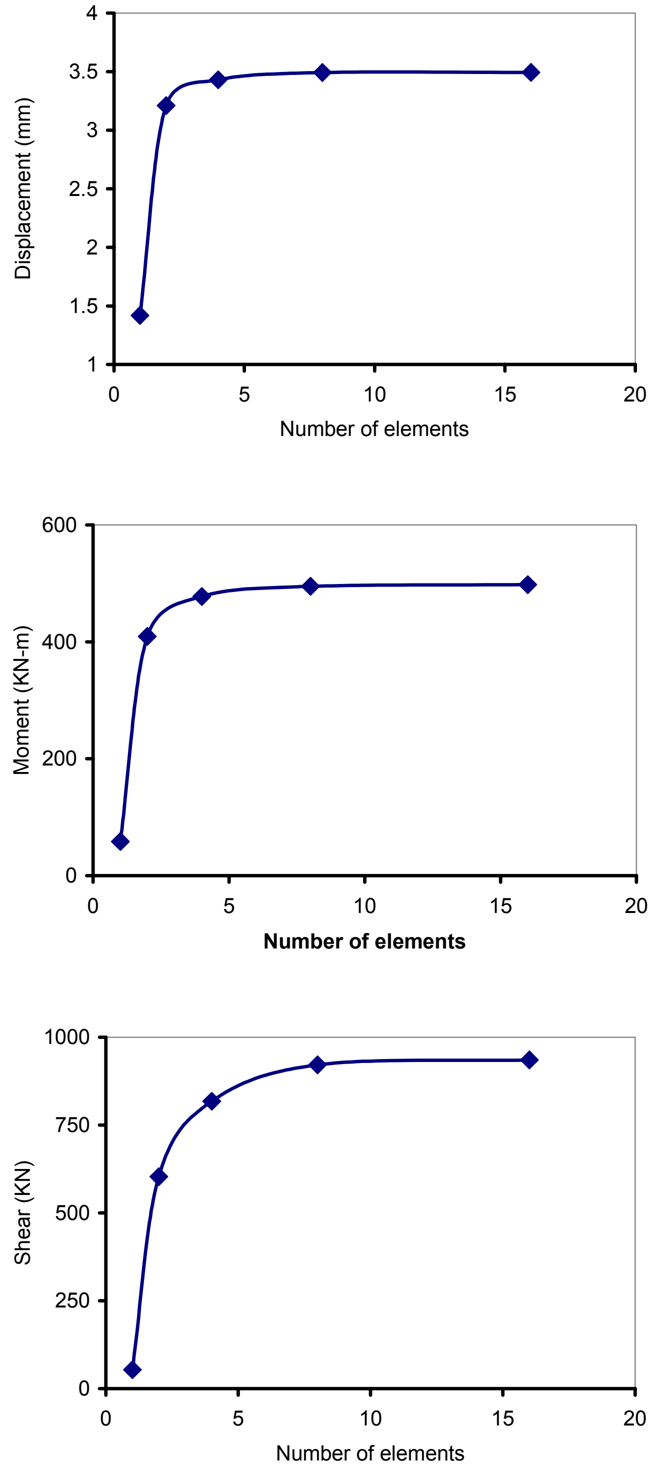

Figure 1: Convergence of the solution for a circular arch under point-load at the apex and self-weight. (a) Displacements at the apex; (b) Moments at the apex; (c) Shear at the apex.

The two groups that investigated the behavior of Cathedrals also had to look at the context in which such churches were built, including why they had that form, why arches were used in the construction. Only one student had traveled to Europe and visited cathedrals in Italy and Spain, and became the referent as a direct witness. They were given the main dimensions (height of the main nave 33m, width 15m) but decided on the loads, supports and member dimensions. Then they modeled a cross section of the church using frame elements.

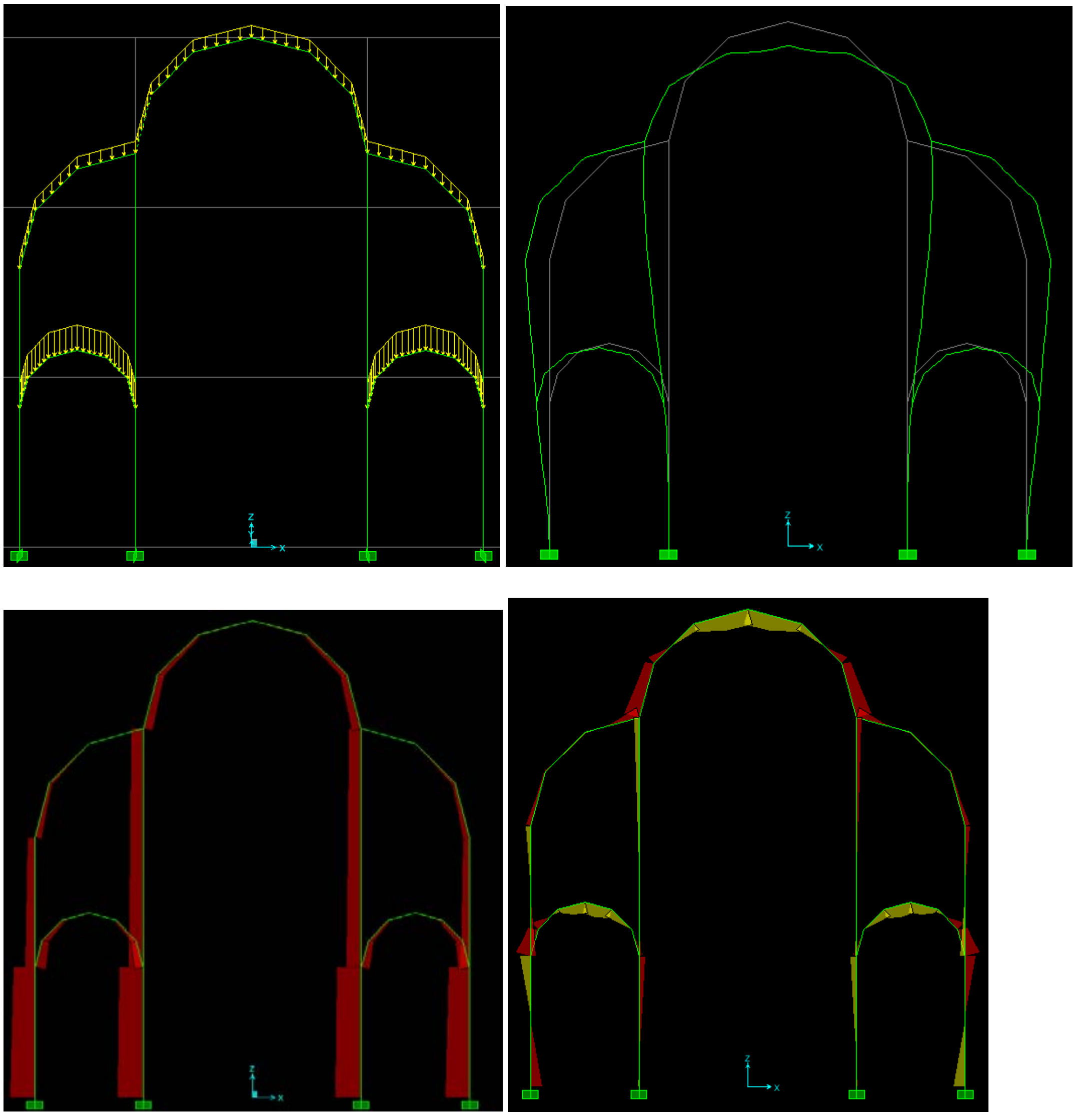

Figure 2: The Romanesque Cathedral, (a) Loads, (b) Deflections for the first model using slender members; (c) Axial forces for the second model with strong elements; (d) Bending moments for the second model.

The results for the Romanesque Cathedral are given in Figure 2, with a central nave, an ambulatory nave and a tribune in the second floor. The first attempt was made using present-day concepts, with slender columns; from that they obtained an extremely flexible structure, with large deflections and tension all over the place. After consultation with the instructor, they changed the dimensions and obtained a much better solution. They could identify what members in the church were vital to produce equilibrium, and why they had to be robust. This group and the next one generated a lot of interest in the class during the presentation, and many comments were made by their colleagues.

The group that modeled the Gothic Cathedral used the same overall dimensions for the central nave and designed a church with architraves and eliminating the tribune. They understood why there was such a change in architecture and the motivations to have more natural light into the central nave. The same sequence was followed: in the first attempt they obtained a flexible design, with large deflections. Then, they modified the dimensions to have a more robust design and computed a reasonable solution in terms of displacements and lack of tension. Some results are presented in Figure 3. Notice that the group assumed simple supported boundary conditions at the bottom, mainly because they had previously designed a steel frame in a Steel Design class, and thought that this was the only condition in practice. Finally, because they knew that there were tensors sometimes added to such churches, they had the initiative to include the influence of a tensor linking the top of the columns in the central nave. They discovered the importance of the architraves in carrying forces to the external part of the church.

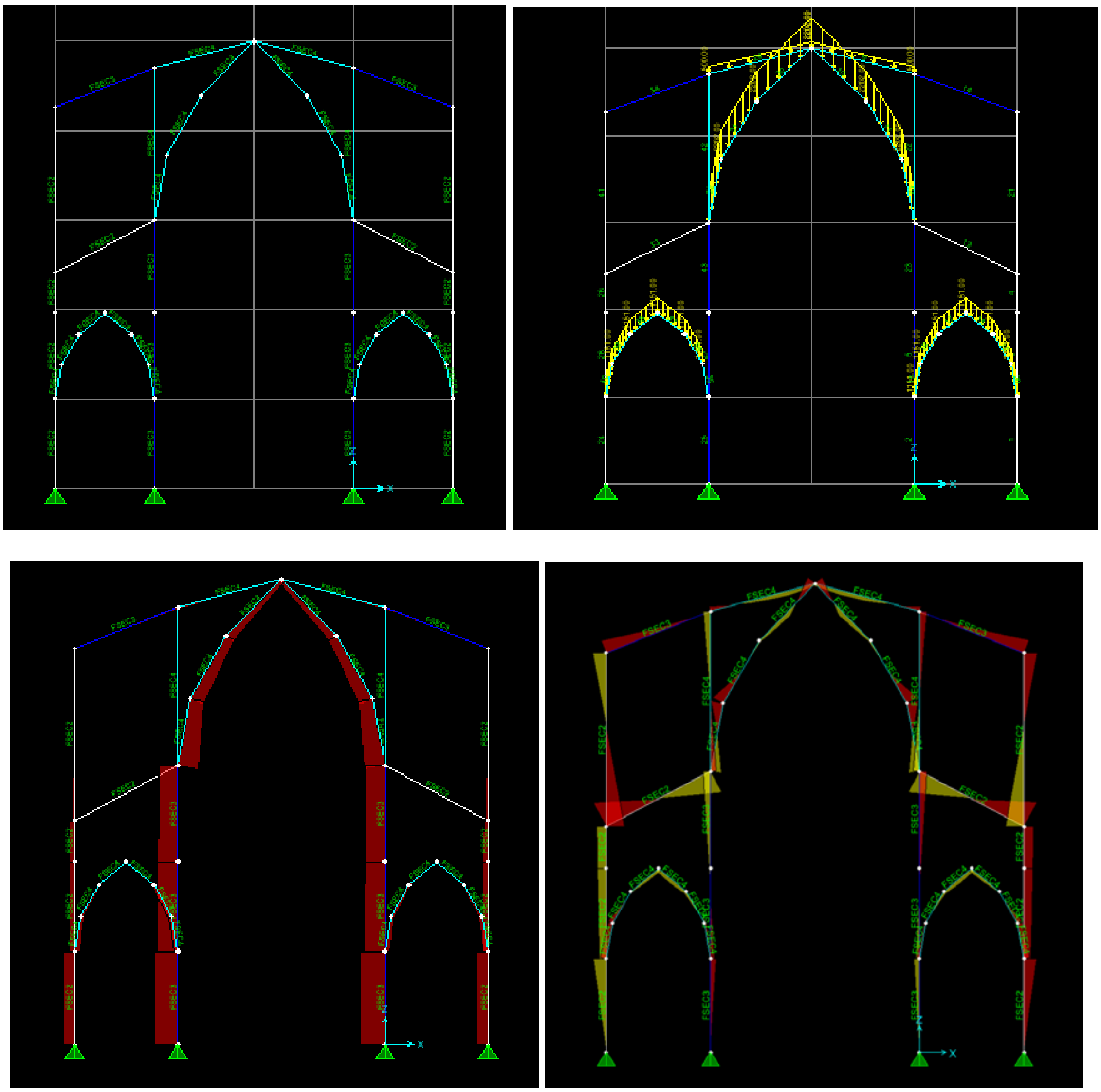

Figure 3: The Gothic Cathedral, (a) Frame idealization, (b) Loads; (c) Axial forces for the second model with strong elements; (d) Bending moments for the second model.

The projects related to bridges were also extremely interesting, One group studied a small wooden bridge (Figure 4.a) following the example of a bridge constructed at West Virginia University at the beginning of the 90s. Another group investigated the historical scenario when the Salginatobel Bridge, in Switzerland, was constructed, and produced a structural analysis using the dimensions that they could get from the Internet. There was a long discussion about the scaffolding constructed for the bridge, shown in Figure 4.b.

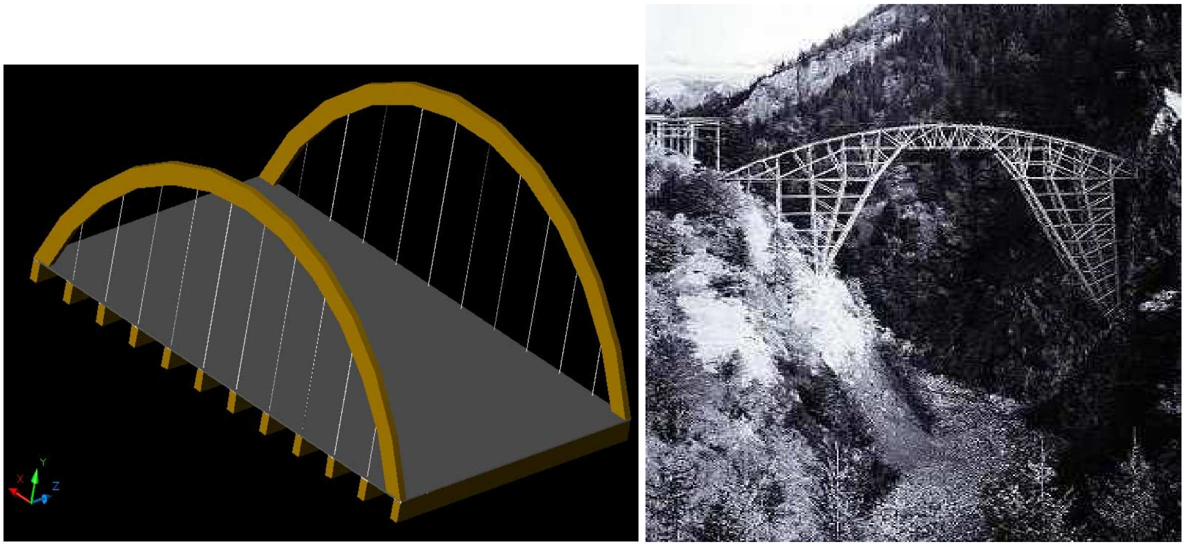

Figure 4: Projects related to bridges: (a) Small wooden bridge analyzed by one of the groups. (b) Scaffolding of a concrete bridge (Salginatobel Bridge, Switzerland, 1930) modeled by one of the groups.

Evaluation of the instruction

A questionnaire was responded by the students after the presentations of the projects in class, to have an initial assessment of the impact on the learners. The main results were:

1. 81% answered that they learned much better the use of the software thanks to the project; 19% reported an intermediate improvement; and 0% did not improve thanks to the project.

2. For 94% of the students, the projects were very successful in introducing a historical dimension about the evolution of structures; for 6% there was an intermediate success; while 0% considered that a historical dimension was not introduced.

3. 94% considered that they learned a lot about arches (now they know about the structural behavior of an arch); 6% had an intermediate understanding, while 0% had little understanding (they now know very little about the structural behavior of an arch).

4. The students identified that in this project, 81% had to reason about how to model the structure; 81% had to imagine the structure they were analyzing; 69% had to search for information on the Internet; 88% had to try several alternatives before they got what they wanted; 63% had to develop skills they did not have to use before in the course.

5. For 31% it was easy to model an arch using a computer program for frames; for 69% the task had intermediate difficulty; and for 0% it was very difficult.

6. For 50% the project demanded a lot of work; for 50% intermediate work; for 0% it was easy to do, demanded little work.

7. The fact that the textbook does not cover arches was a big difficulty for 19%; an intermediate difficulty for 44%; and 37% did not feel a special difficulty.

8. For 88% this activity was more fun than a normal class; for 12% it was similar to a normal class; and for 0% it was boring.

9. 94% would recommend this activity to their peers with enthusiasm; 0% would not recommend the activity; and 6% had an intermediate answer.

Conclusions

This paper reported a strategy to teach arches in Structural Analysis courses in the civil engineering curricula, in which the arch is modeled using frame elements. This is felt to be a reasonable compromise, since it does not require the development of a special technical theory, but it allows exploring the coupling between membrane and bending effects in the arch. As such, arches serve also as an introduction to more complex shell structures.

Examples are given of work carried out by students at the University of Puerto Rico, in which arches were modeled by means of a computer program for frames (SAP 2000) to gain understanding of the behavior, evaluate discretization errors, and carry out sensitivity analysis. The students felt that this was a highly motivating activity, in which they learned not just about arches, but also about their context of historical development. The students said that there was some problem with not having the topic covered by the textbook, although it was not crucial for their task. The effort demanded by the projects was categorized between big and intermediate. Most of the students (about 90%) enjoyed the activity more than a normal class, and would recommend it to other students with enthusiasm (94%).

It is expected that this positive experience may motivate other instructors to include arches in their Structural Analysis courses.

References

[1] Anonymous (2002), SAP 2000: Integrated Software for Structural Analysis and Design, Computers and Structures Inc., Berkeley, CA.

[2] Binding, G. (2002), High Gothic: The Age of the Great Cathedrals, Taschen, Koln, Germany.

[3] Henry-Claude, M. et al. (2001), Principles and Elements of Medieval Church Architecture in Western Europe, Editions Fragile, Gavaudun, France.

[4] Hibbeler, R. C. (1999), Structural Analysis, Fourth Edition, Prentice Hall, NJ.

[5] National Geographical Society (1992), The Builders: Marvels of Engineering, NGS Press, Washington DC.

[6] The Natural Bridge and Arch Society (2003), http://www.cyberseek.com/nabs/

Acknowledgements

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Appendix

The following table contains a summary of the Structural Analysis books that were examined in this research to identify the importance given to arches.

Title	Author	Edition/Year Pages	Publisher	pp. arches	% arches	Contents
Fundamentals of Structural Analysis	K Leet, C-M Uang	2002 pp. 712	McGraw Hill	14	2.0%	Isostatic
Structures	D. Schodek	2001 pp. 576	Prentice	27	4.7%	
Elementary theory of structures	Y-Y Hsieh, S T Mau	4 Ed., 1995 pp. 356	Prentice	0	0	n/a
Introduction to Structural Analysis and Design	S D Rajan	2001 pp. 639	Wiley	1	0.16%	Definition
Analysis of Structural Systems	J F Fleming	1997 pp. 546	Prentice	0	0	n/a
Analysis and Behavior of Structures	E C Rossow	1996 pp. 694	Prentice	0	0	n/a
Structural Analysis	A Kassimali	1995 pp. 697	PSW	6	0.8%	Isostatic Examples of trusses
Fundamentals of Structural Analysis	H. H. West LF Geschwindner	2002 pp. 543	Wiley	4	0.7%	Isostatic
Structural Analysis and Behavior	F Arbabi	1991 pp. 554	McGraw Hill	23	4.15%	Chapter Indeterminate
Elementary Structures for Architects and Builders	R E Shaeffer	3 ed, 1998 pp. 388	Prentice	0	0	n/a
Structural Analysis	A Chajes	1990 pp. 444	Prentice	7	1.5%	Chapter Isostatic
Structural Analysis	R Hibbeler	3 Ed, 1997 pp. 716	Prentice	12	1.7%	Chapter Isostatic
Structural Analysis	R Hibbeler	4 Ed, 1999 pp. 566	Prentice	8	1.4%	Chapter Isostatic
Structural Analysis	R Hibbeler	5 Ed., pp.	Prentice			Chapter Isostatic
Structural Modeling and Analysis	C. Dym	1997 pp. 248	Cambridge	3	1.2%	Definition
Structural Analysis	A Ghali AM Neville	1997 pp. 743	E & FN Spon	5	0.7%	Influence lines
Matrix Methods of Structural Analysis	MB Kanchi	2 Ed., 1993 pp. 521	Wiley	0	0	n/a
Structural Analysis	HI Laursen	3 Ed., 1988 pp. 408	McGraw-Hill	10	2.5%	Chapter (+cables) Isostatic
Structural Analysis: A Classical and Matrix Approach	JC McCormac JK Nelson	2 Ed., 1997 pp. 573	Addison-Wesley	10	1.7%	Isostatic
Introduction to Structural Mechanics and Analysis	DA Dadeppo	1999 pp. 423	Prentice	0	0	n/a

Introduction to Structural Mechanics and Analysis	DA Dadeppo	1999 pp. 423	Prentice	0	0	n/a
Computer Assisted Structural Analysis and Modeling	M Hoit	1995 pp. 405	Prentice	0	0	n/a Cover picture is an arch!!
Análisis Estructural	O González Cuevas	2002 pp. 581	Limusa	0	0	n/a

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Prof. L. A. Godoy graduated as a Civil Engineer at the National University of Córdoba, Argentina, and received his Ph. D. from University College London (University of London, UK) in 1979. He is Full Professor at the University of Puerto Rico at Mayagüez since 1994, and is also Adjunct Professor at West Virginia University. Godoy has published two books: Thin Walled Structures with Structural Imperfections (Pergamon Press, 1996), and Theory of Elastic Stability (Taylor and Francis, 2000). His research interests include engineering education, structural stability, granular flow, computational mechanics, and plates and shells, and results of his research have been published in more than 100 peer-reviewed journal papers.