Developing Student Electromagnetic Intuition

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Abstract

This paper presents the construction details of five demonstrations intended to provide an eyes on and hands on intuitive link to the concepts of the electromagnetic force, superposition, and geometric implications of Maxwell's equations. The vector calculus operators of Maxwell's equations are described in terms of common concepts of atmospheric pressure and bolt action. Intuitive links between the equations and the right hand rules are presented. The mathematics and mechanisms are each presented with practical links to the sometimes faster access memory, intuition. Initial student responses indicate that the cost effective and relatively low power demonstrations add both to the enjoyment of study and the retention of the calculus concepts involved.

Introduction

The information age, more than presenting a vast knowledge base to many is itself rapidly using more wireless methods to connect its clients and disseminate its content. Especially with respect to the Internet, this combination of trends requires the speed of an intuitive grasp of electromagnetic concepts, both to fit new technologies into one's own understanding, and to rapidly recognize some of the nonsense published therein. The unregulated nature of the Internet is in marked contrast to the seemingly highly regulated nature of Nature [1], and a relatively short intuitive study of Maxwell's equations will serve the student well in this arena.

Intuition, as defined in the Thorndike and Barnhart Dictionary, is "perception of truths, facts, etc., without reasoning". Even though today's vast knowledge base requires seemingly limitless specialization of occupation or vocation (scientist, engineer, technologist, etc.), as reasonable humans, one might suspect any perception "without reasoning", even for the sake of speed. However, today's society is as competitive as it is vast, and accurate intuition often has that specific advantage; therefore, a balance between the intuitive and the calculus may help navigate this current technological age with greater end result.

The success of Mr. Albert Einstein's tensor equation relating what one "feels" as 8π times what one "measures" may be the most striking example of the simplicity of "Nature herself". His achievement, more than explaining the failure of electromagnetic theory to predict the photoelectric effect, reminds us to look for the simplest regulation of nature. Nevertheless, in some sense, his general relativity equation ascended to this level on the previous work by Lorentz and Maxwell. [2]. Professor Richard P. Feynman himself wrote that the Maxwell's equations were "relativistically ready".

It is useful to share with students that these representations of the nature of the electromagnetic forces are more tedious than esoteric and still extremely useful today. Indeed, using the electromagnetic force to communicate over miles with microwatts [3] or launch in less than 600 feet, aircraft from ships with megawatts [4] requires only the counting of the electrons motion in space and time – and the Lorentz and Maxwell's equations provide the mechanisms.

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The Mathematics

When rubbed with a woolen sock, certain plastic hairbrushes attract and repel bits of paper, demonstrating the basic concepts of charge and electric force. The magnetic force can be demonstrated (carefully in these days of Neodymium) using disk head positioning magnets held in student's hands. To further demonstrate the concept of the derivative with respect to time and the geometry of the curl, requires only waving these same magnets past coils of wire connected to small flashlight lamps, pointing out the relative speed and orthogonality required to illuminate (or destroy) the lamp filament. These clearly relate what one feels (force) to what is measured (distance, time, volts, and amps). Moreover, these intuitive beginnings can be coupled to the tedium of calculus for a more complete and useful education. The balance achieved affects the ease with which one commands charge, whether the charge moves to plate silverware, communicate with Mars, or convert the pluck of a guitar string to one hundred decibel sound.

The Lorentz Force and Maxwell's Equations

In differential form the equations are: [2]

1. $F = \rho(E + v \times B)$ This is the Lorentz force Law. This is some of the force to conserve in transformers, or be noisy and wear out faster, the magnetic force on moving electrons. This force is the force of the rubbed plastic straw attracting paper. It is instructive to mention to first year students that this charge is measured by counting 6.25 x 10^{18} electrons, give or take many billion, or one Ampere-second in our common units.

2. $\nabla \times E = -\frac{\partial B}{\partial t}$ The curl of electric force is minus the time rate of change of the magnetic force. Wave the single pole of a permanent magnet past a coil of wire soldered to the ends of an incandescent lamp. Note that the shape of a coil of wire carrying current that could replace the Neodymium. Most of our domestic electricity is generated in coils of wire with magnetic force fields circling through them. One seeks the thinnest insulation possible for magnet wire to fit the most coils in the curl.

3. $\nabla \times H = J + \frac{\partial D}{\partial t}$ The curl of the magnetic force is the flow of charge plus the time rate of change of the electric force. This is the traveling radio wave relationship and antenna elements are often oriented in the direction of the D vector. The magnetic field of the common electromagnet is the sum of circular flux lines around each electric current segment.

4. $\nabla \bullet B = 0$ The divergence of the magnetic force is zero. Enclose any source of magnetic force within a surface and no net flux will escape that surface. The cumulative shadow of this vector is zero. A magnetic force line segment cannot be created without returning to its own ends. One might be tempted to say that no magnetic monopole exists, but just as Sir Isaac Newton's Second Law of motion [7] was modified by the relativistic nature of the motion of charge specified by Mr. Maxwell. Mr. Maxwell's equations were again modified by Mr. Einstein. To say something does not exist in Nature would certainly challenge a repetitively reversing history.

5. $\nabla \bullet D = \rho$ The electric force diverges from the point of charge. A point charge at the center of a sphere will create the electric force vector orthogonal to every tangent plane of that sphere. Unlike the magnetic force, these force vectors will not return to a single charge.

Certainly, these equations will appear "Greek" to the new student, and indeed many western nations acknowledge Greece for the symbols and the Pythagorean orthogonal calculation of the geometrical relationships of the forces. By first relating the operators to common perception and then reminding the student that the calculus involved is merely tedious, some aversion to study may be avoided. The del, dot, and cross product operators can be described intuitively in many ways [5]. The following examples, though by no means exhaustive, are often within the common student experience.

The del operator (∇) operates on a field to produce the gradient of that field. Pilots and SCUBA divers operate in a field of air and water influenced by their densities and force of gravity. Reducing the vector gradient descriptions of space and force to one direction is still quite useful to both professions and to the intuitive speed of recognition of the major component in various configurations. With respect to up and down only, the pilot knows that for every thousand feet he climbs, the pressure of the air falls by about 1 inch on the mercury barometer. He learns instinctively to adjust everything from his control rates to fuel mixtures to accommodate altitude. The diver knows that for every 30 feet or so of descent, his ambient water pressure doubles. The successful diver further knows to integrate that over the time of his dive to accommodate the increased fluid and gaseous pressure within his body and avoid the effect of the bends upon ascent back to the surface.

In both cases, reminding the student that the differential calculus involved is merely a shortcut for subtracting and dividing in that one direction, the values of pressure versus altitude is a powerful antidote for the natural aversion to study new (foreign) concepts. Make the case that subtracting itself is a shortcut for counting (down) and division is a shortcut for subtraction, and we teach these concepts to children, but also point out that ε (permittivity) and μ (permeability), the apparent constants with respect to the electric and magnetic force gradients, are orders of magnitude higher than the one-half pounds per square inch (psi) per foot, and one inch of mercury per thousand feet above sea level of the water and atmospheric air gradients. The fact that ε and μ are factors of the speed of light c, points out that the moving charge creates an orthogonal wake. That wake will quickly leave the earth even if the relative speed of charges' motion is merely sleight of hand (as in a handheld experiment). Finally, for both the electric and magnetic vector fields, although they are measurably affected by the matter with which they share space, that effect is accounted for in relative multipliers (of ε and μ) assigned by measurement of that intervening matter.

The dot product and cross product are well described mathematically in the literature [5]. However, it often seems helpful to describe them in terms of the orthogonal shadow of one vector upon another. The screw action of the torque can be perceived as the force vector "crossed" with the wrench vector upon the common bolt. The key concept to note about the cross product is that it yields a new vector whose direction is orthogonal to both the original vectors being crossed.

No inductive coupling occurs between static electric and magnetic forces. The partial derivatives in the equations assert that speed becomes the scaling factor [6]. Modern disk drive magnets can burn out a small incandescent lamp if accelerated near a 300-turn coil wired to the bulb.

The Mechanisms

The modern student is accustomed to stimulating visual processes. It does not impugn his study discipline to include physical and tactile demonstrations to aid his development. Indeed, it is possible to construct electromagnetic demonstrations safely and cost effectively with reasonably dramatic results. Magnet wire and core materials are readily available.

The demonstrations for this paper are:

1. The open air inductive coupling of two windings.
2. The superposition of the magnetic force vector of two orthogonal windings.
3. Resonant repulsion of non-magnetic conductors: Extending the magnetic vector.
4. Eddy current heating and resonance in magnetic conductors.
5. Creating ionizing voltage levels from 12 volts with transformer action.

Open Air Inductive Coupling of Two Windings

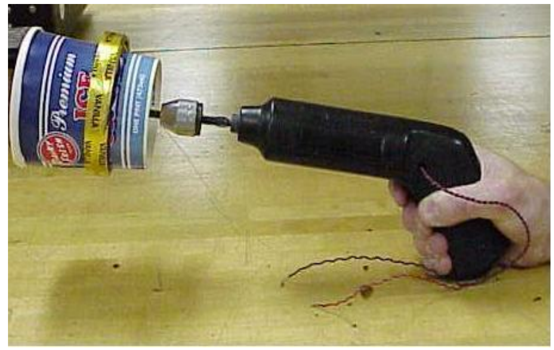

Figure 1. A simple winder and form.

To demonstrate the geometry of the curl, the experiment involves winding coils, driving one coil, measuring the voltage or current induced in the other coil, and varying the orientation of the coils in space relative to each other. Coils of #26 - #28 AWG magnet wire wound on simple forms to produce planar circles are sufficient (Figures 1, 2, 3). Equipment necessary to complete the requirements to demonstrate the orthogonal relationships inherent in the descriptive equation,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

would include an oscilloscope and a laboratory signal generator of typically 50 ohms output impedance capable of supplying a 5 to 10 volts sine wave from 1 to 10 kHz. Simplifying the magnetic force vector to a short line segment centered within a thin ring of wire turns, can be quickly and intuitively grasped as the sum of the many curls of magnetic force rings about each straight segment of the conducting wire coils. The negative sign reminds one that induced currents reduce the magnitude of the magnetic force.

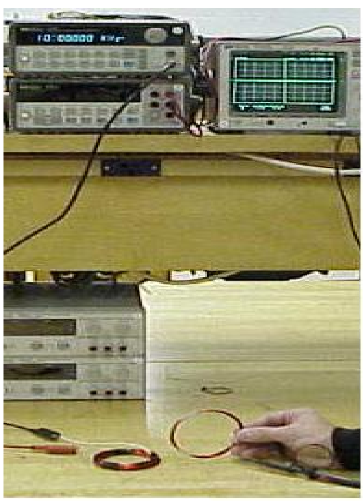

Figure 2. No coupling.

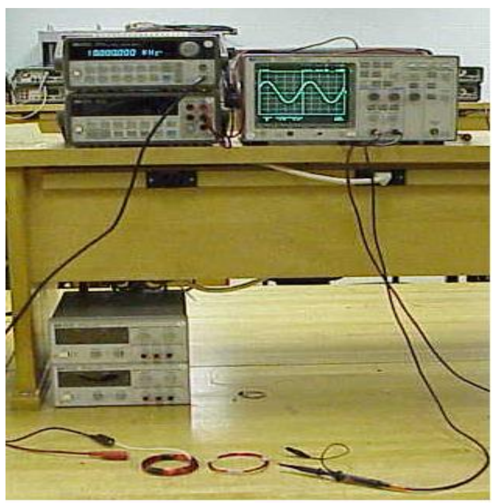

Figure 3. Greater coupling

Figures 2 and 3 depict the oscilloscope displays respective to the relative positions of the driving and the sensing coils. Note that even greater signal pickup would be achieved if the pickup coil was positioned directly above the driven coil, and further, various materials could be inserted between to test their effect on the magnetic induction. Especially dramatic is the insertion of low loss, high permeability magnetic materials (ferrites) within the center of co-linearly aligned coils.

The Superposition of the Magnetic Force Vectors

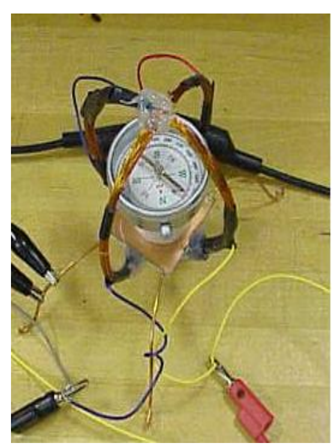

Figure 4.Two orthogonal vectors.

Although it may not be intuitively obvious at first glance, the electric and magnetic forces at any point in space are the result of many sources. In fact, any study of relatively weak magnetic force must deal with the extra component sourced by the earth itself. A dime-store compass that can be read to even five degrees is precise enough to demonstrate the superposition concept and display the mechanisms of vector addition.

The setup of Figure 4 with the 100 turn coils positioned as shown will be able to overcome the earth's magnetic field and demonstrate the superposition of two orthogonal vectors adding up to one resultant vector at currents of about 0.5 Ampere.

By instructing students to put to use the trigonometry previously learned, they may calculate the current required in each coil to force the compass needle to any arbitrary position. For each coil and direction of positive current, the right hand rule may be used to predict the magnetic vector direction. The vectors can be summed to demonstrate the concept of addition of vectors in the four quadrants of the plane. The compass needle yields to the superposition, with visible result.

Figure 5 depicts the waveforms used to drive the needle as a two-phase synchronous motor rotor. Note that two signal generators are required, and each must be capable of sub hertz frequencies and supplying signals of such phase stability that their relative phase may be adjusted by slight momentary changes in one of the generator's frequency setting. This may require some practice.

Figure 5. A phase relationship for rotation.

Resonant Repulsion of Non-magnetic Conductors: Extending the Magnetic Vector

Granted that the curl of electric force will yield a magnetic force orthogonal to the plane of the curl, and a line of magnetic force changing in time will induce a curl of electric force – the Lorentz force law predicts that force can be manifested in the conductors themselves. Since the magnetic force is scaled by materials having a higher permeability than air, a cylindrical core can be constructed to demonstrate the range of motion achievable with common, high permeability, low conductive loss materials. Choosing magnetic materials such as ferrite, which have low conductivity, and hence low eddy current losses, makes levitation possible with such currents available in the typical undergraduate laboratory. One such a linear core was constructed by stacking a number of three-quarter inch (outside diameter) toroid cores and wrapping them in paper. The particular toroids were chosen for their ability to accommodate a conductive but nonmagnetic ring made by cutting out the top of an aluminum soft drink cap.

For this demonstration the primary drive coil was not constructed, but purchased and used "as-is". This coil was a quarter pound spool of #18 AWG magnet wire left on its plastic spool. A 2.2 microfarad film capacitor rated at 200 Volts was wired directly across the coil terminals to increase the circulating current magnitude. The linear core was inserted directly into the center hole of the spool and the aluminum ring slipped over the linear core. A transistor is used to amplify the output of the signal generator to the one or two Amperes necessary to provide demonstrative force. Adjusting the frequency to match the resonance of the coil and capacitor combination illustrates several aspects of the magnetic induction required for lift, namely the cores will audibly vibrate, the aluminum ring will heat to uncomfortable temperatures if held down against the lift force, and the resonant frequency will achieve the greatest lift (Figure 6).

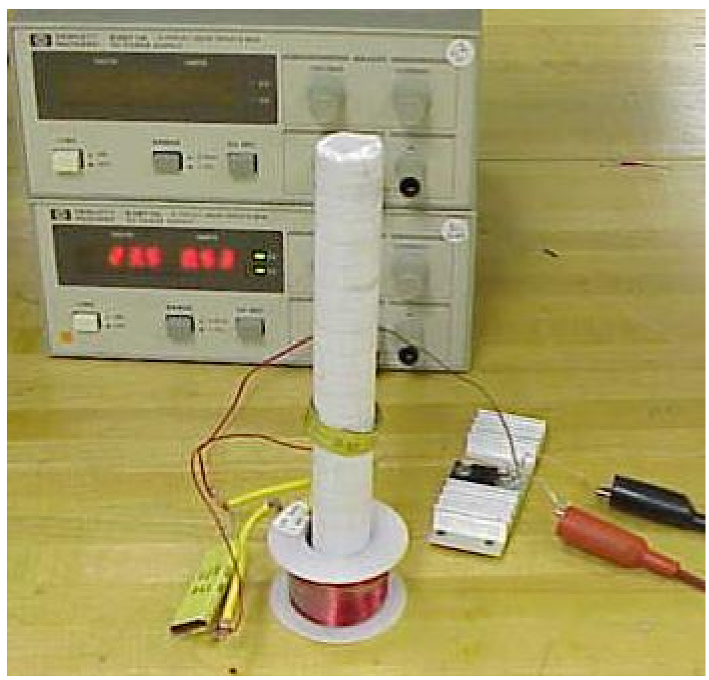

Figure 6. Resonant Levitation.

It may be pointed out to the students that the sound comes from the varying magnetic force against the ferrite, the lift from the Lorentz force, and the heat from the induced currents in the aluminum ring. This particular setup could not excite the ring to fly off the linear core without dipping the aluminum ring in liquid nitrogen directly before slipping it over the core. The decreased resistance of the cold aluminum made the demonstrative difference.

Later modifications of the setup included using a microphone and amplifier of three transistors to drive the primary coil. This allowed students to "whistle lift" the ring on the core and conduct further experiments with remote pickup coils and speaker amplifiers to hear the magnetically coupled transmission of their voices over some three or four meters distance through air space. It is imperative to remind students that this, and even the first experiment, are indeed the essence of all radio communication, and that the Federal Communications Commission regulates all such transmissions in these United States.

Eddy Current Heating and Resonance in Magnetic Conductors.

To quite dramatically demonstrate the acoustic force generated by a time varying magnetic force in mild steel, a foot long piece of 1" diameter rebar rod can be modified to balance on a centered fulcrum, such that it may resonate lying stably on its side. This can be accomplished by taping a small box capacitor at the center of the length of rod (Figure 7). Use the same signal generator and transistor current amplifier to drive a new coil wound on thin-wall PVC pipe of 1.25 inch diameter (Figure 8). This coil consisted of 177 turns of #16 - #18 AWG magnet wire closely spaced along ten inches of the plastic pipe. This pipe's inside diameter will accept the insertion of the 12 inch long scrap of 1" diameter rebar rod with its modified fulcrum.

Figure 7. Construction site scrap of rebar.

Figure 8. Heating or singing steel.

Adjusting the signal generator to 15 kilohertz with the rod only halfway inserted into the coil will demonstrate eddy current heating. Students should be reminded that the iron rod is a resistive conductor and therefore will experience induced circular currents in that portion within the time varying magnetic force field within the coil. After just 5 minutes within the 15 kilohertz field at 10-15 watts excitation, remove the rod and let students feel the temperature difference of the two halves (Figure 8).

To demonstrate the acoustic result of the time varying linear magnetic force on the magnetic rod, it should be inserted it as shown while paying specific attention that it is balanced upon its fulcrum. Adjust the frequency of the signal generator driving the transistor wired to the coil and dc power source to resonance. Note that the sound produced could damage human hearing. Students equipped with appropriate hearing protection may confirm that it is indeed the rod vibration creating the shrieking sound by damping the exposed end with a finger. For the particular rod of approximately 12 inches length the resonant frequency was 7930 hertz. Note that the "Q" of the rod is so high that one must adjust the frequency in tenth hertz steps to achieve the maximum effect.

Creating Ionizing Voltage Levels from 12 Volts withTransformer Action.

One use of electromagnetic coupling that is quite ubiquitous and essential to the technologist is the ac transformer. It is useful because it can transform one volt-ampere product to another; the transformer fails in time from heat and the mechanical forces upon its windings and magnetic coupling cores. A 1000-turn coil can be wound and used for further understanding of electromagnetic intuition by demonstrating the voltage gain of transformer turns ratio. This coil must be on a form that will let the linear core slide within, and the result slide into the 1.25-inch diameter, coil of 177 turns used in the resonance and heating demonstration (Figure 9).

Figure 9. The sliding clearance.

Such a coil may be constructed by hand in less than a half hour, by using the spacing of a thumb to wind successive wraps of 100 turns. Ten such repetitions, linearly spaced, should match in length the 177 turn coil. Magnet wire of #26 AWG on file folder paper used as a form will suffice.

The same signal generator and transistor current amplifier was used to drive the 177 turns coil (Figure 10). The signal generator was adjusted from 400 hertz to 15 kilohertz to point out to students the variation in current draw and output voltage versus that frequency. The student was reminded of the negative sign in the $\partial B / \partial t$ term in the "curl of E"; this specifies that the resultant magnetic force is less within the same volume as a result of the induced electric current in the secondary. The perfect transformer then has no net magnetic flux.

Figure 10. Gas discharge illumination.

Figure 10 shows such a transformer illuminating a 6" fluorescent lamp. Additionally, experiments shorting the 1000-turn coil leads and drawing them apart will draw an arc up to a few millimeters. Note that input voltages greater than 12 Volts will yield enough energy to burn skin, thin paper, etc.

Figure 11 depicts the transistor current amplifier used in conjunction with the laboratory signal generator and dc power supply to drive the ac coils. Note that the transistor must be properly installed on a heat sink.

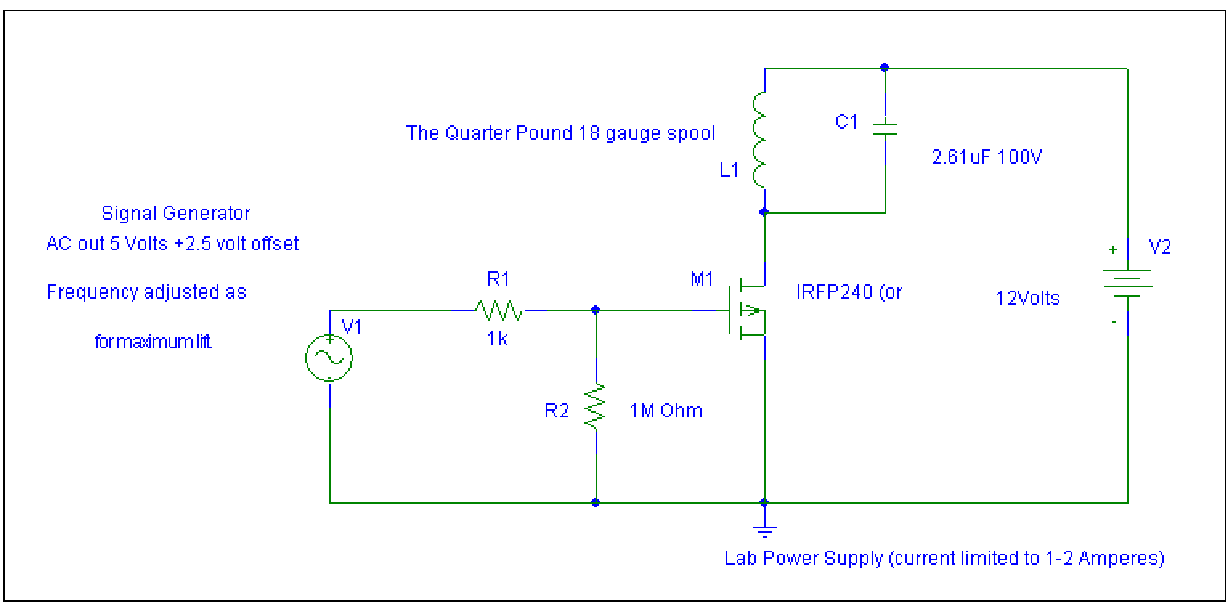

Figure 11. Primary drive circuit.

Conclusion

One full semester using the five demonstrations led to many more. A second semester noted an increased interest, but the school had also an increased enrollment, and they appeared to be more aggressive about learning. Most student feedback indicated that many more examples of actual numbers and units of the electromagnetic force in Ampere turns and meters or feet are necessary to complete a well rounded intuitive grasp of the magnitudes and geometries involved. However, students did report that actually sensing these forces in the single plane and line setups, along with the advantage of noting specific voltages and currents in real time have given them a much better intuitive grasp of just how much force a few dozen Ampere-turns per inch is, and what relative directions the electric and magnetic forces are. Significantly, students of the vector calculus reported that using simple planar coils and considering only the straight line magnetic field produced and coupled to another planar coil, enhanced their intuitive grasp of the cross product relationship. Several students have since brought various commercial motors and automobile alternators for further study, and significantly, asked for help in visualizing the resultant magnetic vectors from the electric coils in more complex geometries.

Finally, one is reminded that the prediction of units and geometries is a mere tedium of the calculus, but the final fact that the cross product is not commutative between vectors is profound indeed. The arbitrary assumption of the label of charge determines that there is a North and a South related for positive current flow in the direction of the fingers of the right hand if North is the orthogonal thumb. Today's technologists, engineers, and physicists alike may use a "left-hand-rule" for predicting the magnetic field resulting from electron flow – knowing full well the arbitrary nature of the labels and the profound concept of the orthogonal nature of the electromagnetic coupling.

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Graduated Tau Beta Pi from Tulane University in New Orleans (BSEE) in 1971, Dan went to work as apprentice to engineers automating domestic fuel pipelines and U.S. Navy and domestic ship propulsion systems. After becoming a registered professional engineer, (#16588 in Louisiana) he developed patented electronic control systems for Pellerin Milnor Corporation's continuous batch washers and embedded PID controls for the company's line of million BTU per hour commercial dryers. As consultant to industry he has designed electronic control and monitoring systems for the U.S. Navy, motorcycle racing enthusiasts, and home automation companies. His current interests are his Flirtoculars™ and designing and building a reasonably nice looking garden robot that will keep a beam of sunlight shining all winter's day into his north office window.