Applying Modern Tools in a Traditional Dynamics Course

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Abstract

A traditional lecture course in engineering dynamics is enhanced by an EPBL* insertion: a trebuchet design/build/test challenge. While other trebuchet build/test projects have been reported, this learning experience differs by requiring ***students to first derive the dynamical solution and then predict performance prior to*** construction and proof-testing. While the analysis applies first principles, the problem (transient 2D motion of a system of three rigid bodies) is complex enough to warrant the application of modern tools by each student. Specifically, the system of kinematic and kinetic relations is solved symbolically using Maple®. Then a predictive tool of the transient motions is created in Excel®, complete with macros and custom functions programmed in Visual Basic for Applications®. The paper describes the development and delivery of the EPBL insertion, results and feedback and instructors' recommendations for future improvement.

**Experiential problem-based learning:* The outcome of a 'learner centered' classroom process that uses real work problems to motivate students to 'perform' or 'act out' the discovery and application of concepts. By doing so, students develop *familiar* as well as *formal* understanding of course content.

Background

Engineering Dynamics is offered to all second year mechanical, civil and industrial engineering students of the College of Engineering and Computer Science at UT Chattanooga. The learning objectives of the three credit hour course include knowing and applying the dynamical principles of impulse & momentum, work & energy, kinematics and kinetics for particle and planar rigid bodies. The course also seeks to add depth to student proficiencies in the application of modern engineering tools, particularly Maple®, Excel® and Visual Basic for Applications® (VBA).

The method of course delivery generally follows the traditional teacher-centered lecture/exam paradigm requiring pre-lecture student preparations and post-lecture reinforcement. Pre-lecture student preparation is promoted through graded classroom recitations where students are *randomly* selected to solve pre-assigned problems at the board. Post-lecture reinforcement is advanced through the daily assignment of homework and through a mandatory correction policy. This policy, based on the premise *"one learns from one's mistake"*, compels students to correct the mistakes contained in all graded work by rewarding perfect corrections with a 50% recovery of any loss points. To manage the correction policy, students are required to submit an end-of-semester portfolio containing all work and corrections.

To maximize the likelihood that the course learning objectives are achieved, a strategy to supplement the otherwise traditional lecture course is employed through *insertions* of experiential problem-based learning (EPBL) projects. EPBL is the outcome of a learner-centered process that uses real work problems or scenarios to motivate students to perform or act out the discovery and application of concepts and information. By doing so, students develop *familiar* as well as *formal* understanding of course content.[1] [2]

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Evolution of the Trebuchet EPBL Project

The current trebuchet learning activity evolved out of a series of *spring-loaded catapult* EPBL projects first developed in 1999, modified in 2000 and again in 2001[3]. These out-of-class projects required students to apply the principles of projectile motion, particle kinetics, work & energy and impulse & momentum in the design, construction and proof testing of spring-loaded catapults that launched tennis ball. The earliest of these projects were team-based with no individual deliverables and were delivered to the class in the form of a contest or team challenge to obtain the longest launch distance. Subsequent catapult projects were broken down into two phases and included both individual and team deliverables for grading. In phase 1 students were individually responsible for developing and submitting his or her analysis and preliminary design of a spring loaded launcher in accordance with specified constraints. In phase 2 teams were formed, each responsible for selecting, enhancing, constructing then proof-testing a launcher. The last round of the spring loaded catapult project added the requirement that Maple® would be used to derive and solve the system of kinematic and kinetic relations.

While entertaining, the spring-loaded catapult problem was not very challenging. This conclusion was based on the observation that the design analysis was limited to basic particle kinematics and kinetics and contained none of the thornier applications of rigid body dynamics. Furthermore, the application of Maple® to solve the system of linear equations was a trivial task easily accomplished by hand without the use of a solver. The EPBL project was therefore again revised in spring 2002 with the introduction of the Floating Arm Trebuchet.

The Floating Arm Trebuchet

The trebuchet is a medieval gravity powered engine of siege warfare designed to hurl a projectile, such as a heavy stone or a diseased animal, at a distant target such as castles. The idea of inserting a trebuchet into the dynamics course was suggested by a student who completed the spring 2001 dynamics course. Knowing nothing about medieval siege engines, a web search found a plethora of medieval siege engine sites and trebuchet enthusiasts, both academic and hobbyists[4,5,6]. Searches of the educational literature revealed reports of trebuchet learning projects used in an engineering course[7,8,9]. The trebuchet appeared to be a good candidate for an EPBL insertion into the engineering dynamics course because it would require knowledge and application of general planar rigid body kinematics and kinetics and because it could be constructed with readily available materials.

FAT Version 1.0 Spring 2002

The initial FAT EPBL project was delivered to the class in the spring of 2002 as an individual challenge where each student developed a design with supporting analysis to predict trebuchet performance and as a team contest where each team selected, optimized and constructed a FAT with the longest throwing distance. At that time, the instructor (erroneously) believed that the dynamical solution to the trebuchet *and sling* applied the method of Lagrange and its extension to holonomic constraints[10], approaches beyond the scope of this second year dynamics course. Therefore the beta version of the FAT *problem* did not contain the sling. Thus the analysis of the motion of the throwing arm and projectile could be estimated without advanced tools of analysis.

The project schedule was divided into two phases that corresponded to individual and team-based activities and deliverables. In Phase1 all students proposed his or her design solution supported by a substantially complete dynamical analysis. The dynamical analysis included concept sketches, properly labeled free body diagrams, derivations of governing equations of rigid body kinematics and kinetics, solutions to the system of dynamical equations using Maple®, and a functional Excel design tool that could be used to predict the throwing distance and optimize any of several parameters to maximize that distance. In Phase 2, students were assigned to teams of 3 to 4 members who evaluated their respective design proposals then selected one as the team's preliminary design. Teams then enhanced that design with sufficient detail for construction and reported the predicted performance. On acceptance of the report by the instructor, the teams constructed the prototype then delivered the FAT for demonstration and proof testing.

The project specifications called for a gravity powered floating arm trebuchet with a counterpoise, a throwing arm or lever, trigger release mechanism and a freestanding wood support structure. The throwing arm articulated on two slider-mounted axels where one pair of sliders was constrained to translate vertically and the other translated horizontally. Referring to the free body diagram in Figure 1, points B and C indicate the vertically and horizontally constrained axels, respectively; the counterpoise hangs from point A and the projectile is seated at point D.

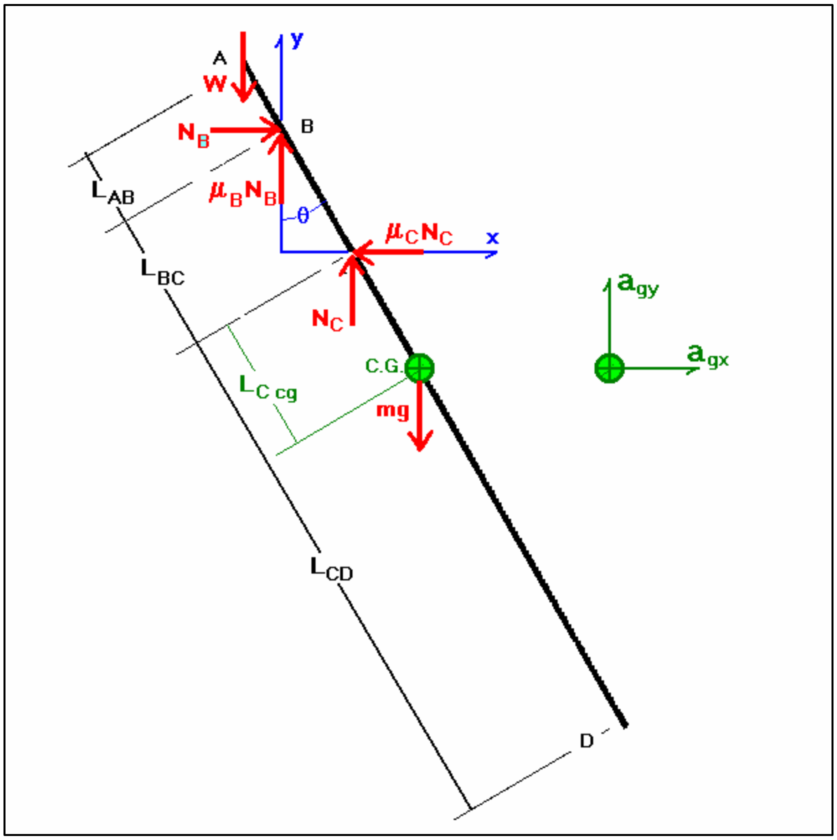

Figure 1 Free body diagram of Floating Arm Trebuchet (less sling) shown in loaded position. On trigger release of counter poise W, sliders at B and C move vertically and horizontally, respectively, while projectile at D moves in an elliptical path until released.

The Student Deliverables for Phase 1 Design Proposal included:

- Concept sketch that described all elements, centers of mass, supports, loads and dimensions and labeled with appropriate symbols.
- A set of complete and properly labeled free body diagrams with the corresponding set of derived kinetic and kinematic relations
- Maple® worksheet containing the solution of the system of equations: kinetic equations of motion and kinematic equations of general plane motion
- Excel® worksheet containing plots of:
    - Angular acceleration vs. angular position
    - Angular velocity vs. angular position
    - Projectile Range vs. angular release position

The Project Specifications called for the use of Maple® to solve the system of equations in order to estimate the angular velocity ω as a function of angular position θ. The closed form solution for angular acceleration α was readily obtained in Maple. This solution, however, was not trivial as it contained over 5000 characters and spaces exceeding the 1024 size limit for an Excel formula. Therefore custom functions were created in Visual Basis for Applications

(VBA) by copying then pasting the expression from Maple to the VBA editor where each line held the same 10-bit size limitation.

While the closed form expression for acceleration was derived in Maple, that for angular velocity was not. Therefore velocity was obtained by a numerical integration of angular acceleration using Excel and macros using VBA. Excel worksheets with custom functions and macros were also designed by students to predict projectile range as a function of user-defined values for counterpoise, lever and projectile masses, critical lever dimensions, coefficients of friction, and initial angular position.

Roughly half of the students submitted satisfactory individual design proposals on their first submittal. All but one of the remaining students satisfied the content requirements on their second submittal. Unexpectedly, the instructor did not have sufficient time to properly review, debug and grade the individual analyses and Excel worksheets. The *plan* was to assess the goodness of each student's design by comparing it directly to the instructor's design template. However, suffering a total lack of imagination, this instructor did not anticipate the numerous ways a free body diagram could be culled together. Thus the design template poorly accommodated the multitude of differences in FBD labels, dimensions and coordinate systems of the students' work. Therefore, the feedback to individual students was limited to: "Good: Submittal is complete. Results are generally consistent with instructor's"; "Satisfactory: Submittal is complete. Results are not consistent with instructor's"; and "Unsatisfactory: Incomplete submittal".

In Phase 2 three teams were assigned and designs were finalized. Due to time limitations, the team design report requirement was waived and construction was fast-tracked. Two teams built the trebuchets on campus in the engineering machine shop. The third catapult was constructed off campus. The specified time and date for delivery of the functioning trebuchets for proof testing coincided with the scheduled date and time of the final exam. The class met at an outdoor playing field. In spite of the overcast damp weather, many curious onlookers gathered including dozens of students, faculty, staff as well as the local radio, television and print media outlets. For the shootout, teams test fired the trebuchets by hurling the official projectile, a baseball. Volunteers shagged fly balls, measured and recorded distances. Although one team encountered numerous mechanical problems eventually corrected with a substantial quantity of duct tape, the other catapults operated reliably. Once all the requisite test measurements were made and recorded, the students remained on the practice field for well over an hour, to show off their catapults, to hurl more objects, to chat with curious onlookers, to take pictures, to play, to enjoy the moment. How closely did the actual performance match the predicted? Not closely at all. The maximum observed range for "*sling-less*" catapults measured roughly 15 m while the predicted range was 42 m. It was reasoned, at the time, that the deviation was due the absence of a means to "arrest" trebuchet motion at the optimum angular position, a shortcoming observed in all three team designs.

Student feedback was acquired informally through group and one-on-one interactions. Again, most students agreed that the EPBL insertions increased their interest in and their perceived relevance of the course materials. The most recurring dissent pertained to the amount of out-of-class time consumed completing the analysis. Several others expressed frustrations using Maple® and programming VBA and Excel®. One student opined that trebuchet construction should be an individual deliverable. Several, after complaining about the time consumed developing, debugging and refining their analyses, pointed to the inadvertent learning results: they could readily derive the equations of motion and kinematics; they could solve the system of equations in Maple®; and they were able to create Excel® macros and custom functions in VBA! Finally, most expressed an appreciation for the opportunity to demonstrate their handiwork and expertise on the campus practice field to a curious and impressed audience. The formal student evaluations of the spring 2001 course were unusually high with median scores of 6.0 on a scale of 6.0 in three of four categories: instructional quality, course content and interest in subject matter. The accompanying student comments reiterated the positive and negative feedback cited above.

The initial floating arm trebuchet EPBL project was successful. With regard to the maximizing the likelihood of achieving learning objectives, the trebuchet challenge turned out to be a good EPBL project choice because it reinforced the *formal classroom knowledge* of 2D rigid body dynamics with the *familiar knowledge* obtained through immersion in the solution of the real, hands-on problem. With regard to enhancing competencies in the application of modern engineering tools, the EPBL project was also a good choice in two ways. First, it exposed students to a sufficiently complex problem where the use of a symbolic solver such as Maple® the application of a numerical method in Excel and VBA was warranted.

FAT Version 2.0 Spring 2003

While the initial FAT project proved successful, student feedback and instructor observation suggested there was much room for improvement. Therefore in the spring of 2003, the FAT EPBL project was further enhanced in both content and delivery.

The content was expanded to required analysis of a FAT trebuchet *with a sling* but without reliance on advanced methods of analysis. Instead, fair predictions of system behavior would be obtained by coupling the motion of sling as a sliding then free swinging rigid pendulum onto the kinematics of the counterweighted throwing arm. Regarding delivery, student complaints and frustrations related to the time demand and the learning/relearning of Maple and Excel was addressed by breaking down Phase 1 (the design process) into a closely guided sequence of tasks, spread out over the entire semester. Each of the tasks described in the sequence required an individual deliverable:

1. Given a set of relevant constraints, using particle kinematics and the calculus of extrema, derive expressions for maximum projectile range in Maple and plot the maximum range (as a function of release angle) in Excel.
2. Given a set of relevant constraints, draw the free body and kinematic diagrams for the trebuchet system and each of its components: treb arm with sliders, counterpoise and sling with projectile.
3. Given a set of relevant constraints, derive the system kinematic and kinetic equations of motion for the trebuchet system in Maple® then solve that system for the angular accelerations of the treb arm α_{arm} and sling, α_{sling}.
4. Given results in 3, develop in Excel and VBA, a numerical integration to estimate the instantaneous velocity of the projectile as a function of treb arm position.
5. Incorporate the result in 1 with that in 4 into an Excel *design tool* predicts the performance of a floating arm trebuchet with sling based on a set of user-defined parameters.
6. Complete a study to optimize critical parameters to maximize hurling distance.
7. Prepare and submit preliminary report.

Grading of the preliminary reports was expedited because each of the preceding 6 tasks had already been checked. Reports were simply check for agreement with the instructor's solution and for completion then marked: "Good: Submittal is complete. Results are generally consistent with instructor's"; "Satisfactory: Submittal is complete. Results are not consistent with instructor's"; and "Unsatisfactory: Incomplete submittal". After the preliminary reports were graded, and four teams were formed and Phase 2, construction and proof testing, was kicked-off. Teams finalized designs and constructed prototypes on campus in the engineering workshop shop. Delivery for testing the functioning trebuchets was scheduled for date and time of the final exam. The class met at an rain soaked outdoor playing field for test firing.

How closely did the actual performance match the predicted? More closely that the previous year, with three of four falling roughly 15% to 20% shy of predictions ranging between 32 to 45m. The fourth treb suffered serious mechanical problems related to an undersized and severely deformed axel. The deviations from the predicted performance were attributed to the coarse and primitive construction, to a lack of control on the sling release angle and to unaccounted aerodynamic effects.

Again, most students agreed that the treb project increased their interest in and their perceived relevance of the course materials. And again, most complained about the amount of out-of-class time consumed completing the project. Several again pointed positively to their new found expertise in Excel, VBA and Maple. The formal student evaluations of the spring 2002 course were again high with median scores of 5.5 on a scale of 6.0 in three of four categories: instructional quality, course content and interest in subject matter.

Conclusion

Trebuchet design task reinforces the principles of particle and 2D rigid body kinematics and kinetics. The predictive tasks enhance competencies in the applications of Excel, VBA and Maple. The construction and testing generate a tremendous amount of student interest, involvement, enthusiasm and satisfaction. For these reasons, the trebuchet challenge is an excellent EPBL project choice for insertion into any undergrad engineering dynamics course.

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