Analytical and Numerical Methods: A Beneficial Combination

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Abstract

Numerical and analytical mathematics subjects have been incorporated into a single junior level course entitled "Engineering Analysis for Mechanical Engineers." The course meets specific needs of the curriculum and is intended to raise the mathematical abilities of the students in preparation for late junior and senior level engineering courses. The rationale, design, and anticipated benefits of the course are presented.

Although not usually taught together, both analytical and numerical methods in engineering fall under the category of applied mathematics. Classical advanced engineering mathematics textbooks such as that by Wylie and Barrett[1] or by Kreyszig[2] devote most of their attention to background mathematics and analytical methods but do not forget to address numerical methods. In many engineering curricula, however, these subjects are taught separately perhaps due to the conceptual differences. The two areas are actually analogous in many ways and offer benefits when taught together.

Background and Motivation

When Mercer University transitioned from a quarter to a semester system in the Fall of 1997, the School of Engineering took this opportunity to implement a significant round of changes in its engineering curricula. Total program hours were reduced from 132 semester hours to 128 semester hours, and, in anticipation of the next accreditation visit occurring under EC2000, a variety of other curricula changes were introduced.

Among these changes were several adjustments to the sequence of mathematics and mathematics related courses traditionally taken by engineers. The freshman level course in numerical methods was dropped from the curriculum along with the requirement that all engineering students take multivariable calculus. In addition, engineering students were placed in a different mathematics sequence than other math/science majors for the two courses in differential and integral calculus. Engineering students took a two course pair—a (3-0-3) course [3 lecture hour - 0 lab hour- 3 total credit hours] taught by the math department and a (0-3-1) course [0 lecture hour – 3 lab hour – 1 total credit hour] taught by engineers for the engineering students versus a single (4-0-4) course [4 lecture hour – 0 lab hour – 4 total credit hours] taught by the math department for math/science majors] for each of these two basic calculus courses. The changes also altered some of the basic course content, not only in the calculus courses, but also in the introduction to differential equations class in that, because of the math sequence changes for engineers, the math department viewed the engineers as having a weaker background entering differential equations than did other students.

Fallout from some of these changes became apparent after the first year of the new curriculum. Students entering introductory second year classes seemed to have little (or, more accurately, no) concept of

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algorithmic problem solving or of the limitations of numerical schemes in solving particular problems. Additionally, engineering student capabilities in mathematics, at least as observed in the thermal science sequence of the mechanical engineering specialization, appeared to suffer as students exposed to the new math sequence reached their junior year courses. While, admittedly, this latter issue had been an ongoing problem for a number of years, the sequencing change coincided with an apparent exacerbation of difficulties associated with mathematics competency.

During the second year of the semester program, based on student performance, the recommendation of curricular consultants evaluating the relationship of engineering and math/science programs, and the desire of several engineering specializations (including mechanical engineering) to return the requirement of a full course in multivariable calculus, the mathematics segment of the curriculum was again revised. Under this revision (in effect at the current time) engineering students and math/science students take the same two calculus courses—both taught in a (3-0-3) course taught in mathematics and a (0-3-1) course organized and taught by both mathematics and engineering faculty.

In order to attempt to address the remaining problems related to mathematics and numerical methods competency for mechanical engineering applications and to solidify the mechanical engineering specialization's basic science and mathematics component, a new mechanical engineering course was proposed. In principle, this course was designed to provide a roughly five week introduction to the solution of partial differential equations using the method of separation of variables and to provide a ten week development, from an essentially mathematical perspective, of the background, assumptions, and limitations associated with the use of standard numerical techniques in mechanical engineering.

The course was included in the first term of the junior year to continue the curriculum's mathematics thread through the junior year and right up to the point of the most significant use of upper level mathematics in the Mercer mechanical engineering specialization—fluids mechanics, heat transfer, thermal systems design, and engineering systems analysis.

Course Organization

Students are introduced to several topics in applied mathematics considering both analytical and numerical methods used in the analysis of engineering problems. The course is strictly applied mathematics as opposed to 'pure' mathematics, and as mentioned above, is meant to be a step between the mathematics traditionally taught in the freshman and sophomore years by a mathematics department and senior engineering courses applying advanced mathematics. One intent is to promote a better understanding of the analytical basis for much of the formulas and solutions which are at the disposal of senior students, graduate students and even practicing engineers.

The prefix analytical could refer to any of a tremendously broad range of mathematical topics. From this perspective, the analytical methods considered here are severely limited, but specifically chosen. Fourier analysis, the characteristics of parabolic, elliptic and hyperbolic partial differential equations and the separation-of-variables solution technique constitute the analytical component of the course. These topics form a sufficient background in partial differential equations in preparation for upper-level engineering courses and, in particular, heat transfer.

The numerical methods segment considers topics taken selectively from each of the fundamental areas which are quite standard in most numerical-methods courses for engineers. The topics include: sources of numerical errors, approximation of differential operators, solution techniques for simultaneous equations (linear and nonlinear), root finding, curve fitting, integration, and differential equations. An introduction to finite difference techniques in two dimensions concludes the treatment of numerical methods with which the instructor could re-address one of the previous problems solved analytically to encourage a more incisive understanding.

A course outline of the topics covered is presented below. Approximately one third of the course lectures are devoted to selected analytical topics with the remaining devoted to numerical methods. The numerical methods topics marked with (link) indicate a reference back to the earlier coverage of analytical methods.

ANALYTICAL (Farlow)
Analysis motivation/ Problem solving strategy
Introduction to partial differential equations
Fourier Analysis
Parabolic - Unsteady Heat Equation (1D)
Hyperbolic - Wave equation (1D)
Elliptic - Laplace's equation (2D)
Parabolic - Unsteady Heat Equation (2D)
Hyperbolic - Wave equation (2D)

NUMERICAL (Chapra and Canale)
Introduction to Numerical Methods (Concept of Modeling)
Computer Representation of Numbers
Sources of Numerical Errors
Finite Difference Approximations
Linear System Solution techniques (Direct and Iterative)
Singular Systems and Matrix Condition
Root Finding (link)
Nonlinear Systems
Curve Fitting (Least Squares Regression, Polynomial Interpolation)
Fast Fourier Transform (link)
Numerical Integration Techniques
Solution Techniques for 1st Order ordinary differential equations
Systems of ordinary differential equations
Finite Difference Methods for partial differential equations (link)
Adv. and Disadv. of Numerical and Analytical solutions (link) (Farlow)

A search for a single textbook sufficient for both subject areas was unsuccessful. Thus, the course is taught with reference to two textbooks. Partial Differential Equations for Scientists and Engineers by Stanley Farlow[3] covers the analytical topics with the exception that more attention is given to Fourier analysis than that of Farlow.[3] Numerical Methods for Engineers by Chapra and Canale[4] is used to teach the selected numerical techniques of the course. Neither textbook is covered in its entirety. Attention is focused on specifically chosen fragmented sections throughout each text.

Other than the obvious links between a few of the numerical methods topics and particular analytical topics covered prior, three class lectures attempt to take advantage of teaching analytical and numerical techniques together. The introductory lecture presents the need for understanding analytical, numerical and experimental techniques and describes one possible collaboration of these techniques in a problem solving strategy. The purpose of this lecture is to distinguish these three categories of techniques, show the similarities, and explain the values, limitations and roles that each might have at analyzing an engineering problem. Later, the introduction to the numerical methods portion describes the steps of modeling as in Chapra and Canale,[4] beginning with the mathematical model of the physical problem which will include simplifications, then the numerical model of the mathematics with unavoidable truncation errors, and finally the simulation of the numerical problem with unavoidable round-off errors. The final lecture is designed to discuss the advantages and disadvantages of analytical and numerical methods in light of what the students have learned.

The course utilizes the mathematical software package, Mathcad®, particularly chosen because of the following features: 1) a convenient and intuitive graphical user interface, 2) many built-in numerical methods functions, 3) elementary programming capability, 4) easy visualization of numerically and analytically represented functions, and 5) symbolic manipulation capabilities. During the analytical portion of the course, students use the graphing and symbolic manipulation features primarily for verification and visualization purposes of analytically derived solutions. Only limited programming is required of the students during the coverage of numerical methods. Most assignments require the application of a numerical methods by hand which can be checked with Mathcad®.

Anticipated Student Benefits

The individual topics discussed above are not new to engineering programs and possess recognized merit not to be reiterated here. It is expected that an understanding of the individual topics listed above would benefit undergraduate and graduate students and practicing engineers. In addition to valuable topics, the course attempts a new avenue for a broad perspective with the described combination of numerical and analytical topics under the concept of analysis. Students are exposed to analysis and numerous problem-solving techniques, and gain experience using mathematical tools to describe and solve physical problems.

The analytical category of topics is intended to provide students with an advanced understanding of partial differential equations (PDEs) in a minimal number of course hours. Applying techniques like variable transformations, separation of variables, similarity techniques, etc. which may be used in senior engineering courses should now require less discussion regarding background mathematics affording additional time to consider the engineering aspects of course material. Furthermore, students should gain insights regarding when analytical solutions might be attainable and when numerical methods and/or experiments become preferred.

The numerical category of topics will provide a comprehensive view of the computational basis for CAE software. Students should develop some understanding of what goes on ‘behind the scenes’ of computational software in order to address the cause of numerical difficulties that often arise. Students will also gain an ability to apply numerical methods directly by composing a small program, worksheet or spreadsheet providing a degree of freedom of not being completely reliant on CAE software in senior course projects, graduate school and/or engineering practice.

Other than the advantages attributed separately to both categories of topics, there does exists a possibility for "secondary" benefits associated with learning the analytical and numerical topics together. Opportunities will exist to solve common problems analytically and later numerically and to reference earlier material for rationale and in explanations of the current topic. And with the three lectures discussed above regarding analysis in general (the introductory lecture of the course, introductory lecture of the numerical portion, and the final lecture of the course), the students hopefully gain a broad perspective of the relevance, limitations and roles of each of these areas and how skills in all these areas can be combined to analyze and solve engineering problems.

Other secondary benefits are associated with the timing of the course during the curriculum. As an intermediate link between early math and senior engineering courses, information threads exist between topics discussed in calculus and ordinary differential equations (ODEs) and the mathematics applied in senior engineering courses. Each row in Table I below indicates that a thread exists between the listed materials.

Table I. Information Threads

Old Material	Course Material	Senior Material

Calculus	Taylor series	Equation Derivations
	Fourier series	separation of variables
ODE's	separation of variables	Analytical solutions
ODE's	Select ODE's revisited	Analytical solutions
	3 types of PDE's , physical characteristics	Governing equations, Modeling
	Wave eqauation	Vibrations
	Heat equation	Heat transfer
	Laplace's equation	Fluid Mechanics, Heat Transfer
Taylor series	Finite Difference Methods	Course projects, Senior Design, Heat Transfer
	Linear/ Nonlinear Systems Solution techniques	Course projects, Senior Design
	Root Finding	Course projects, Senior Design
	Curve Fitting	Laboratories, Course projects, Senior Design
	Numerical Integration Techniques	Course projects, Senior Design

Observations

Two sections of the course were taught in the Fall of 1999 and one in the Fall of 2000. The students taking the course in the fall of 1999 will complete their cycle of required courses during the fall of 2000. Preliminary indications from spring 2000 and fall 2000 courses do indicate improved cognizance of potential applications and limitations of analytical and numerical procedures. Students also appear to demonstrate an enhanced ability to solve problems whose mathematical basis was introduced not only in this new course both also involving topics introduced in more traditional mathematics courses.

A preliminary search of the mechanical engineering curriculum of many institutions in the ASEE Southeast Section is underway. Many departments teach only the numerical category of topics and very few require the analytical topics and in particular partial differential equations. The Florida Institute of Technology, among those schools requiring the analytical topics, includes a boundary-value-problems course in the mechanical-engineering curriculum and as a separate course teaches a seemingly application-oriented computer-aided-engineering course rather than numerical methods. The mechanical engineering department at the University of Memphis teaches a course entitled "Engineering Analysis" which addresses solely analytical topics including second order ODE's, Laplace transforms, and Fourier series with applications. Some mechanical engineering departments such as at Texas A&M have taught strictly numerical methods in courses entitled "Engineering Analysis."

The combination of analytical and numerical analysis at Mercer University is not unique. For example, the mechanical and aerospace engineering and engineering science department at the University of Tennessee has been teaching a similar engineering-analysis course since about 1995.[5] Their course topics are about two thirds numerical and one third analytical with the analytical portion covering partial differential equations, Fourier series and separation of variables. The numerical methods covered are essentially the same as in the course now taught at Mercer University. One minor difference is that the numerical methods are taught prior to the analytical topics rather than as in the sequence of topics presented above. Both courses face a burden of lacking an appropriate textbook to cover both topic categories. The course at the

University of Tennessee is currently using a numerical methods textbook by Gerald and Wheatley.[6] This marks at least two faculties that independently recognized the need for a such a course in their curricula.

References

[1] Wylie, C. R. and L. C. Barret (1995) *Advanced Engineering Mathematics*, 6 ed., McGraw-Hill.

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[3] Farlow, S. J. (1993) *Partial Differential Equations for Scientists and Engineers*, Dover.

[4] Chapra, S. C. and R. P. Canale (1998) *Numerical Methods for Engineers*, 3 ed., WCB/McGraw-Hill.

[5] Professor J. I. Frankel (2001) Private Communication, University of Tennessee, Mechanical and Aerospace Engineering and Engineering Science Department.

[6] Gerald and Wheatley (1997) *Applied Numerical Analysis*, 6 ed., Addison Wesley.

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