Enhancing the Teaching of Moment Distribution Analysis Using Spreadsheet

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Abstract

This paper illustrates the use of spreadsheet such as EXCEL to enhance the efficiency of moment distribution method. Continuous beam was used to illustrate the concepts. A multi-story, multi-bay framed structure was used to illustrate the efficiency of the method when spreadsheet is used. It was shown that the computer tool reduced the number of random arithmetic errors, student frustrations, and time in doing the tedious computations. Hence the instructors can assign more realistic problems without increasing the time required of the students when compared with the pre-PC era.

Introduction

A half century old numerical method developed by Professor Hardy Cross has, at time, hailed as a major break through to the analysis of complex structures. The method enjoys both mathematical and procedural simplicity. It is one of the classical structural analysis methods taught in the elementary structural analysis course. Although PC-based structural analysis software is plentiful and easily accessible, many civil engineering programs still include this classic, the moment distribution method, in their elementary structural analysis courses.

Despite the simplicity of the method, it has caused the users, both students and instructors as well as engineers, some very frustrated moments. The reason is that the tedious arithmetic often caused a lot of careless mistakes. Unfortunately, these mistakes are hard to find even for a small frame.

The development of spreadsheet gave a much needed enhancement to moment distribution method's efficiency. The objective of this paper is to show how one can set up the spreadsheet to perform the moment distribution analysis. The paper has no intention to replace the students or other users' knowledge of the method. The spreadsheet serves as a "big calculator" whose primary function is number crunching. The user is still required to know the parametric values such as fixed-end moments, distribution factors, carry over factors, and the process of balance-distribute-carry-over. Furthermore, this paper assumes that the readers have a working knowledge of the method.

The basic feature of spreadsheet is that it can perform computation algorithm based on the relative and absolute cell positions of the values involved. Majority of the effort in moment distributions is spent in the process of balance-distribute-carry-over at each joint. And the repetitive nature of the balance-distribute-carry over cycles are all depend on the values from the immediate prior cycles. This lends itself very nicely to spreadsheet format. Once an user sets up the spreadsheet for the first balance-distribute-carry-over cycle, the subsequent cycles can be created by copying the cell formulas from the first cycle. The users, however, have to decide at what cycle should this process be terminated. The ability to copy the cell formulas from cycle to cycle minimizes the unnecessary arithmetic errors. The time and frustration can be directed to the analysis itself.

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In this paper, a 3-span continuous beam was used to demonstrate the moment distribution method using EXCEL. A multi-story, multi-bay framed structure was then used to illustrate the efficiency of the moment distribution method with EXCEL. Summary of survey results from students taught with EXCEL is also included in this paper. Summary of student survey regarding the use of EXCEL with moment distribution is also included.

Moment Distribution Method

Moment distribution method is a classical method that has been included in every civil engineering program. It is one of the first numerical methods for structural analysis. The theoretical concept of the method can be found in any elementary structural analysis text and is well known in the civil engineering profession. Hence, it will not be discussed here. The procedures of the method can be summarized below for prismatic members:

1. Compute the relative stiffness k = I/L for each member that is either connected or supported at ***both*** ends. This excludes overhang members.
2. Compute the distribution factor (DF) for each member connected to a joint or support. For ONE member at a support, the member has a DF = 1.0 if the support is a roller or pin; DF = 0 if the support is fixed. For all members connected to a joint, each member has a DF defined below,

$$DF_i = \frac{k_i}{\sum_{j=1}^{n} k_j} \qquad where\ k_i = \frac{I_i}{L_i}$$, n = number of members at the joint. Note that $\sum DF_i = 1$

3. Compute fixed end moments (FEM) for members with intermediate loads.
4. Compute end moments for overhanging members.
5. Pre-determine a tolerance limit.
6. Balance the moments at each joint and distribute the balancing moment according to DF of each member attached to the joint.
7. Carry over the balanced moment to the adjacent end of each member. The carry over factor for prismatic members is 0.5.
8. Repeat steps 6 & 7 until the unbalanced moments are within the tolerance limit.
9. Sum the moments in each column for each end of a member.

Implementation Using Spreadsheet Software

In this section a 3-span continuous beam is used to illustrate the moment distribution analysis using Microsoft EXCEL software. A 3-story, 3-bay framed structure with 3 joint translations is then presented to illustrated the efficiency of using spreadsheet software for moment distributions.

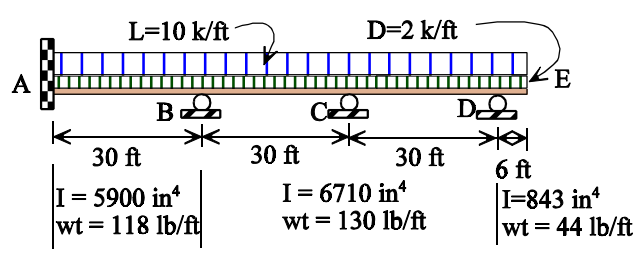

Figure 1. 3-span continuous steel beam subjected to uniform loads and self weight.

Continuous Beam Example

A 3-span continuous steel beam subjected to a uniformly distributed live load of 10 k/ft and dead load of 2 k/ft, and self weight is shown in Fig. 1. In the analysis, it was assumed that the load factors for live load was 1.6 and for dead load and self weight was 1.2. Table 1 shows the EXCEL computation of the analysis,

Table 1. EXCEL worksheet of moment distributions for the 3-span beam shown in Figure 1

	A	B	C	D	E	F	G	H	I
1	member	I (in^4)	L (ft)	k=I/L	D	LL	self	w_u	
2	AB	5900	30	196.667	2	10	0.118	18.542	
3	BC	6710	30	223.667	2	10	0.130	18.556	
4	CD	6710	30	223.667	2	10	0.130	18.556	
5	DE	843	6	140.500	2	10	0.044	18.453	
6									
7	joint	A	AB	BA	BC	CB	CD	DC	DE
8	DF	1	0	0.468	0.532	0.5	0.5	1	0
9	FEM		-1390.62	1390.62	-1391.70	1391.70	-1391.70	1391.70	-332.15
10	balance	1390.62	0.00	0.51	0.57	0.00	0.00	-1059.55	0.00
11	carry over		0.25	0.00	0.00	0.29	-529.77	0.00	
12	balance	-0.25	0.00	0.00	0.00	264.74	264.74	0.00	0.00
13	carry over		0.00	0.00	132.37	0.00	0.00	132.37	
14	balance	0.00	0.00	-61.93	-70.44	0.00	0.00	-132.37	0.00
15	carry over		-30.97	0.00	0.00	-35.22	-66.19	0.00	
16	balance	30.97	0.00	0.00	0.00	50.70	50.70	0.00	0.00
17	carry over		0.00	0.00	25.35	0.00	0.00	25.35	
18	balance	0.00	0.00	-11.86	-13.49	0.00	0.00	-25.35	0.00
19	carry over		-5.93	0.00	0.00	-6.74	-12.68	0.00	
20	balance	5.93	0.00	0.00	0.00	9.71	9.71	0.00	0.00
21	carry over		0.00	0.00	4.86	0.00	0.00	4.86	
22	balance	0.00	0.00	-2.27	-2.58	0.00	0.00	-4.86	0.00
23	carry over		-1.14	0.00	0.00	-1.29	-2.43	0.00	
24	balance	1.14	0.00	0.00	0.00	1.86	1.86	0.00	0.00
25	carry over		0.00	0.00	0.93	0.00	0.00	0.93	
26	balance	0.00	0.00	-0.44	-0.49	0.00	0.00	-0.93	0.00
27	Final	1428.40	-1428.40	1314.62	-1314.62	1675.75	-1675.75	332.15	-332.15

Table 2. Cell formulas for EXCEL computations shown in Table 1.

	A	B	C	D	E	F	G	H	I
1	member	I (in^4)	L (ft)	k=I/L	D	LL	self	w_u	
2	AB	5900	30	=B2/C2	2	10	0.118	=1.2*(E2+G2)+1.6*	
3	BC	6710	30	=B3/C3	2	10	0.13	=1.2*(E3+G3)+1.6*	
4	CD	6710	30	=B4/C4	2	10	0.13	=1.2*(E4+G4)+1.6*	
5	DE	843	6	=B5/C5	2	10	0.044	=1.2*(E5+G5)+1.6*	
6									
7	joint	A	AB	BA	BC	CB	CD	DC	DE
8	DF	1	0	=D2/(D2+D3)	=D3/(D2+D3)	=D3/(D3+D4)	=D4/(D3+D4)	1	0
9	FEM		=-H2*C2^2/12	=-C9	=-H3*C3^2/12	=-E9	=-H4*C4^2/12	=-G9	=-H5*C5^2/2
10	balance	=-(B9+C9)*B$8	=-(B9+C9)*C$8	=-(D9+E9)*D$8	=-(D9+E9)*E$8	=-(F9+G9)*F$8	=-(F9+G9)*G$8	=-(H9+I9)*H$8	=(H9+I9)*I$8
11	carry over		=0.5*D10	=0.5*C10	=0.5*F10	=0.5*E10	=0.5*H10	=0.5*G10	
12	balance	=-(B11+C11)*B$8	=-(B11+C11)*C$8	=-(D11+E11)*D$8	=-(D11+E11)*E$8	=-(F11+G11)*F$8	=-(F11+G11)*G$8	=-(H11+I11)*H$8	=(H11+I11)*I$8
13	carry over		=0.5*D12	=0.5*C12	=0.5*F12	=0.5*E12	=0.5*H12	=0.5*G12	
14	balance	=-(B13+C13)*B$8	=-(B13+C13)*C$8	=-(D13+E13)*D$8	=-(D13+E13)*E$8	=-(F13+G13)*F$8	=-(F13+G13)*G$8	=-(H13+I13)*H$8	=(H13+I13)*I$8
15	carry over		=0.5*D14	=0.5*C14	=0.5*F14	=0.5*E14	=0.5*H14	=0.5*G14	
16	balance	=-(B15+C15)*B$8	=-(B15+C15)*C$8	=-(D15+E15)*D$8	=-(D15+E15)*E$8	=-(F15+G15)*F$8	=-(F15+G15)*G$8	=-(H15+I15)*H$8	=(H15+I15)*I$8
17	carry over		=0.5*D16	=0.5*C16	=0.5*F16	=0.5*E16	=0.5*H16	=0.5*G16	
18	balance	=-(B17+C17)*B$8	=-(B17+C17)*C$8	=-(D17+E17)*D$8	=-(D17+E17)*E$8	=-(F17+G17)*F$8	=-(F17+G17)*G$8	=-(H17+I17)*H$8	=(H17+I17)*I$8
19	carry over		=0.5*D18	=0.5*C18	=0.5*F18	=0.5*E18	=0.5*H18	=0.5*G18	
20	balance	=-(B19+C19)*B$8	=-(B19+C19)*C$8	=-(D19+E19)*D$8	=-(D19+E19)*E$8	=-(F19+G19)*F$8	=-(F19+G19)*G$8	=-(H19+I19)*H$8	=(H19+I19)*I$8
21	carry over		=0.5*D20	=0.5*C20	=0.5*F20	=0.5*E20	=0.5*H20	=0.5*G20	
22	balance	=-(B21+C21)*B$8	=-(B21+C21)*C$8	=-(D21+E21)*D$8	=-(D21+E21)*E$8	=-(F21+G21)*F$8	=-(F21+G21)*G$8	=-(H21+I21)*H$8	=(H21+I21)*I$8
23	carry over		=0.5*D22	=0.5*C22	=0.5*F22	=0.5*E22	=0.5*H22	=0.5*G22	
24	balance	=-(B23+C23)*B$8	=-(B23+C23)*C$8	=-(D23+E23)*D$8	=-(D23+E23)*E$8	=-(F23+G23)*F$8	=-(F23+G23)*G$8	=-(H23+I23)*H$8	=(H23+I23)*I$8
25	carry over		=0.5*D24	=0.5*C24	=0.5*F24	=0.5*E24	=0.5*H24	=0.5*G24	
26	balance	=-(B25+C25)*B$8	=-(B25+C25)*C$8	=-(D25+E25)*D$8	=-(D25+E25)*E$8	=-(F25+G25)*F$8	=-(F25+G25)*G$8	=-(H25+I25)*H$8	=(H25+I25)*I$8
27	Final	=SUM(B9:B26)	=SUM(C9:C26)	=SUM(D9:D26)	=SUM(E9:E26)	=SUM(F9:F26)	=SUM(G9:G26)	=SUM(H9:H26)	=SUM(I9:I26)

and Table 2 shows the cell formulas of the same example problem. In the tables, the first row and first column are the internal column and row headings from EXCEL.

The first 5 rows of the tables are information on member length, moment of inertia and load intensities that were used to compute the relative stiffness and factored loads (step 1 outlined in previous section). Row 7 shows the headings for the ends of each member and supports of the beam. Row 8 shows the DF determined using cells D2 to D5 (step 2). Row 9 shows the FEM for the appropriate members using the loads given in cells H2 to H5 (steps 3 and 4). Row 10 shows the balance-distribute computations for each joint (step 6). Row 11 shows the carry-over moments for each member (step 7). From row 12 on, the cell formulas are similar to those in rows 10 and 11 (first cycle of balance-distribute-carry-over). Cell formulas in rows 10 and 11 were duplicated until the balance-distribute moments were within the tolerance limit set in step 4. Finally, step 9 was executed and shown in row 27.

The intent of this simple problem is to show how a typical moment distribution analysis can be set up. Since the cell formulas in rows 10 and 11 can be duplicated using the *copy-and-paste* commands in EXCEL, random arithmetic errors such as the change of sign during the balance cycle would be eliminated at any one cycle with any one joint. If errors do occur in writing the cell formulas in lines 10 and 11, the numerical errors would be compounded throughout the entire analysis. The final bending moments would have some unrealistic values which can be detected easily. Furthermore, since the students only need to concern with the correctness of the formulas in the first balance-distribute-carry-over cycle, they would tend to be more careful. In the event errors are found, only the cell formulas in the first cycle need to be corrected. The remaining cycles can be corrected by re-copying the revised formulas. Thus, the process eliminates a lot of frustrations arisen from some minor errors that often occurred even to the most experienced engineers. Note that the portion on developing distribution factors (DF) and fixed end moments (FEM) can be done manually with the values inputted in rows 8 and 9 respectively. It is included here for completeness.

Multi-degree Joint Translations Structure

The convenience of using EXCEL or similar software for the moment distribution method can be further amplified in the case of a structure having joint translations. For every joint translation a structure has, a complete moment distributions need to be performed using a new set of FEM due to an arbitrary joint translation. After different cases of joint translations have been considered, the imaginary restraining forces which were used to prevent any joint translations during the moment distribution process must be resolved to satisfy the original constraints and loading conditions of the structure.

Through the commands of *cut-and-paste*, the worksheet for case A (due to applied loads only) can be duplicated. The FEM of case B can then be adjusted to reflect the first arbitrary joint translation induced. If more than 1 joint translations exist in the structure, additional moment distribution worksheet can be reproduced similar to that of case B.

Figure 2 shows the 3-story, 3-bay framed structure subjected to roof live load L_r of 0.125 k/ft, live load L of 1.25 k/ft, wind load W of 0.5 k/ft and P_{W1} of 3 k and P_{W2} of 5 k. For the analysis, it was assumed that the load factor for W, P_{W1} and P_{W2} was 1.3, for L was 1.2 and for L_r was 0.5. The EI for all the columns are the same, and EI for all the beams is 1.5 times the EI of the columns.

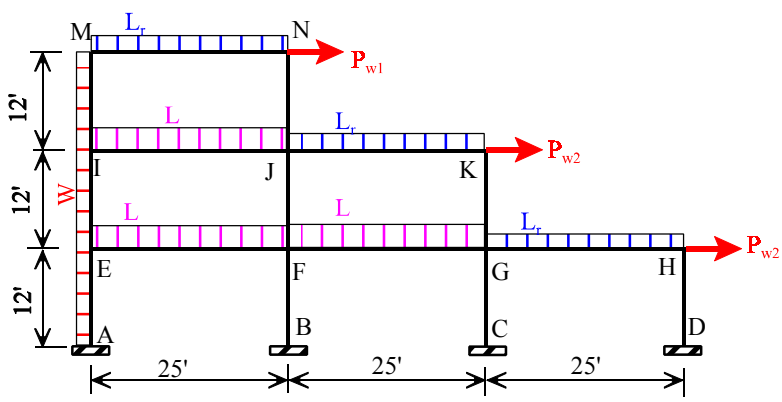

Figure 2. Framed structure with joint translations

Table 3 shows the spreadsheet

computations for case A (due to applied loads only). In the interest of space, the computations of the DF are not presented here. Table 4 shows the spreadsheet computations for case B where joint translation was induced at joints M and N. The entire worksheet for case A was duplicated for case B. The only modification required for case B is the FEM. Since joint translation was induced at joints M and N, only columns labelled as IM, MI, JN, and NJ (labels shown in row 1) have FEM of some arbitrary value, say 100 k-ft. The rest of the FEM are zero. Once the FEM have been revised, the entire spreadsheet for case B will automatically be updated to reflect the new FEM.

Due to the limitation in space, the spreadsheet computations of cases C and D (for joint translations at the middle and bottom levels) will not be presented here but they will be presented in the Conference. As one can see from this example that the amount of work required for structures with joint translations is no longer increased parabolically with the number of different joint translations involved. The tedious work of balance-distribute-carry-over is reduced to one analysis and primarily for one cycle.

If one wishes to continue taking advantage of the capability of EXCEL or similar, the remaining solution can also be "systemized" and solved by EXCEL. The procedures can be summarized below:

1. Compute the shear forces for all the columns for cases A and B. The shear forces for cases C and D are related to the member end moments only in exactly the same way as in case B.
2. Set up equilibrium equation for lateral forces at levels MN, IJK, and EFGH to solve for the multipliers associated with each of the joint translation induced (cases B, C and D).
3. Compute the final member end moments using the linear combination of the moments from cases A through D and the multipliers determined in step 2.

Table 5 shows the layout of the procedures described in steps 1 - 3.

Student Responses

At the suggestions of the reviewers, an email survey was conducted from the students from the fall 2000 class and a few (whose email addresses are still in the file) from the fall 1999 class. A total of 40 students were solicited and 20 responses were received.

About half of the responses (11 out of 20) said that they would have used EXCEL to do the problems even if EXCEL was not presented as part of the lecture. Five students said that they might eventually recognize the advantages of EXCEL and 4 students said that they would not have known that spreadsheet could be a nice tool for the moment distribution method. If the students were given the choice of learning with or without EXCEL, the response was unanimous. They all preferred to learn with EXCEL. However, 8 students indicated that they should use the method by hand first before doing them in EXCEL.

Conclusions

Two examples were presented to demonstrate,

1. how spreadsheet such as EXCEL can be implemented to the arithmetic intense moment distribution method; and
2. how spreadsheet can be used efficiently to solve a complex structure which would have been prohibitively time consuming to assign to the students in the pre-PC era.

The goal of this paper is to expose to those educators as well as engineers who have not fully taken advantage of the capability of the computer tools available to us. After all, one of ABET EC2000 criteria is to demonstrate "*the students' ability to use technique, skills and tools in engineering practice*". This *ability* allows the instructors to assign problems that are more realistic without increasing the demand in students' time. It also gives the students the sense of pride and confidence that even with the knowledge acquired from a fundamental course in structural analysis, they are capable of solving some rather complex structures to completion without using a "black box" type computer software. From the student survey, it is obvious that they prefer to see EXCEL implemented in the moment distribution method.

Table 3. EXCEL worksheet of moment distributions of case A for the 3-story, 3-bay framed structure shown in Figure 2.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1		A	AE	B	BF	C	CG	D	DH	EA	EI	EF	FE	FB	FJ	FG	GF	GC
2	DF	1	0	1	0	1	0	1	0	0.368	0.368	0.265	0.209	0.291	0.291	0.209	0.209	0.291
3	FEM		-7.80							7.80	-7.80	-78.13	78.13			-78.13	78.13	
4	balance	7.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	28.72	28.72	20.68	0.00	0.00	0.00	0.00	-15.67	-21.76
5	carry over		14.36		0.00		-10.88		-0.95	0.00	14.36	0.00	10.34	0.00	-10.88	-7.84	0.00	0.00
6	balance	-14.36	0.00	0.00	0.00	10.88	0.00	0.95	0.00	-5.28	-5.28	-3.80	1.75	2.44	2.44	1.75	0.34	0.47
7	carry over		-2.64		1.22		0.24		2.28	0.00	-0.96	0.88	-1.90	0.00	-1.27	0.17	0.88	0.00
8	balance	2.64	0.00	-1.22	0.00	-0.24	0.00	-2.28	0.00	0.03	0.03	0.02	0.63	0.87	0.87	0.63	-1.67	-2.31
9	carry over		0.01		0.44		-1.16		-0.05	0.00	1.38	0.31	0.01	0.00	-1.15	-0.83	0.31	0.00
10	balance	-0.01	0.00	-0.44	0.00	1.16	0.00	0.05	0.00	-0.62	-0.62	-0.45	0.41	0.57	0.57	0.41	-0.10	-0.14
11	carry over		-0.31		0.29		-0.07		0.24	0.00	0.23	0.21	-0.22	0.00	-0.40	-0.05	0.21	0.00
12	balance	0.31	0.00	-0.29	0.00	0.07	0.00	-0.24	0.00	-0.16	-0.16	-0.12	0.14	0.20	0.20	0.14	-0.20	-0.28
13	carry over		-0.08		0.10		-0.14		0.01	0.00	0.23	0.07	-0.06	0.00	-0.19	-0.10	0.07	0.00
14	balance	0.08	0.00	-0.10	0.00	0.14	0.00	-0.01	0.00	-0.11	-0.11	-0.08	0.07	0.10	0.10	0.07	-0.04	-0.05
15	carry over		-0.06		0.05		-0.03		0.03	0.00	0.07	0.04	-0.04	0.00	-0.09	-0.02	0.04	0.00
16	balance	0.06	0.00	-0.05	0.00	0.03	0.00	-0.03	0.00	-0.04	-0.04	-0.03	0.03	0.04	0.04	0.03	-0.03	-0.04
17	carry over		-0.02		0.02		-0.02		0.01	0.00	0.04	0.02	-0.01	0.00	-0.04	-0.01	0.02	0.00
18	balance	0.02	0.00	-0.02	0.00	0.02	0.00	-0.01	0.00	-0.02	-0.02	-0.02	0.01	0.02	0.02	0.01	-0.01	-0.01
19	carry over		-0.01		0.01		-0.01		0.00	0.00	0.02	0.01	-0.01	0.00	-0.02	0.00	0.01	0.00
20	balance	0.01	0.00	-0.01	0.00	0.01	0.00	0.00	0.00	-0.01	-0.01	-0.01	0.01	0.01	0.01	0.01	0.00	-0.01
21	final	-3.46	3.46	-2.12	2.12	12.06	-12.06	-1.58	1.58	30.31	30.09	-60.39	89.29	4.25	-9.78	-83.75	62.27	-24.13
22																		
23		GK	GH	HG	HD	IE	IM	IJ	JI	JF	JN	JK	KJ	KG	MI	MN	NM	NJ
24	DF	0.291	0.209	0.419	0.581	0.368	0.368	0.265	0.209	0.291	0.291	0.209	0.419	0.581	0.581	0.419	0.419	0.581
25	FEM		-3.26	3.26		7.80	-7.80	-78.13	78.13			-3.26	3.26		7.80	-3.26	3.26	
26	balance	-21.76	-15.67	-1.36	-1.89	28.72	28.72	20.68	-15.67	-21.76	-21.76	-15.67	-1.36	-1.89	-2.64	-1.90	-1.36	-1.89
27	carry over	-0.95	-0.68	-7.84	0.00	14.36	-1.32	-7.84	10.34	0.00	-0.95	-0.68	-7.84	-10.88	14.36	-0.68	-0.95	-10.88
28	balance	0.47	0.34	3.28	4.56	-1.91	-1.91	-1.38	-1.82	-2.53	-2.53	-1.82	7.84	10.88	-7.95	-5.73	4.95	6.88
29	carry over	5.44	1.64	0.17	0.00	-2.64	-3.98	-0.91	-0.69	1.22	3.44	3.92	-0.91	0.24	-0.96	2.48	-2.86	-1.27
30	balance	-2.31	-1.67	-0.07	-0.10	2.77	2.77	1.99	-1.65	-2.29	-2.29	-1.65	0.28	0.39	-0.88	-0.64	1.73	2.40
31	carry over	0.20	-0.04	-0.83	0.00	0.01	-0.44	-0.83	1.00	0.44	1.20	0.14	-0.83	-1.16	1.38	0.86	-0.32	-1.15
32	balance	-0.14	-0.10	0.35	0.48	0.46	0.46	0.33	-0.58	-0.81	-0.81	-0.58	0.83	1.15	-1.31	-0.94	0.61	0.85
33	carry over	0.58	0.17	-0.05	0.00	-0.31	-0.65	-0.29	0.17	0.29	0.43	0.41	-0.29	-0.07	0.23	0.31	-0.47	-0.40
34	balance	-0.28	-0.20	0.02	0.03	0.46	0.46	0.33	-0.27	-0.38	-0.38	-0.27	0.15	0.21	-0.31	-0.22	0.37	0.51
35	carry over	0.10	0.01	-0.10	0.00	-0.08	-0.16	-0.14	0.17	0.10	0.25	0.08	-0.14	-0.14	0.23	0.18	-0.11	-0.19
36	balance	-0.05	-0.04	0.04	0.06	0.14	0.14	0.10	-0.12	-0.17	-0.17	-0.12	0.11	0.16	-0.24	-0.17	0.13	0.17
37	carry over	0.08	0.02	-0.02	0.00	-0.06	-0.12	-0.06	0.05	0.05	0.09	0.06	-0.06	-0.03	0.07	0.06	-0.09	-0.09
38	balance	-0.04	-0.03	0.01	0.01	0.09	0.09	0.06	-0.05	-0.07	-0.07	-0.05	0.04	0.05	-0.08	-0.05	0.07	0.10
39	carry over	0.03	0.00	-0.01	0.00	-0.02	-0.04	-0.03	0.03	0.02	0.05	0.02	-0.03	-0.02	0.04	0.04	-0.03	-0.04
40	balance	-0.01	-0.01	0.01	0.01	0.03	0.03	0.02	-0.03	-0.04	-0.04	-0.03	0.02	0.03	-0.05	-0.03	0.03	0.04
41	carry over	0.01	0.00	0.00	0.00	-0.01	-0.02	-0.01	0.01	0.01	0.02	0.01	-0.01	-0.01	0.02	0.01	-0.02	-0.02
42	balance	-0.01	0.00	0.00	0.00	0.02	0.02	0.01	-0.01	-0.01	-0.01	-0.01	0.01	0.01	-0.02	-0.01	0.01	0.02
43	final	-18.64	-19.50	-3.16	3.16	49.83	16.24	-66.07	68.99	-25.95	-23.53	-19.51	1.07	-1.07	9.70	-9.70	4.95	-4.95

Table 3. EXCEL worksheet of moment distributions of case B for the 3-story, 3-bay framed structure shown in Figure 2.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1		A	AE	B	BF	C	CG	D	DH	EA	EI	EF	FE	FB	FJ	FG	GF	GC
2	DF	1	0	1	0	1	0	1	0	0.368	0.368	0.265	0.209	0.291	0.291	0.209	0.209	0.291
3	FEM																	
4	balance	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5	carry over		0.00		0.00		0.00		0.00	0.00	18.38	0.00	0.00	0.00	14.53	0.00	0.00	0.00
6	balance	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-6.76	-6.76	-4.87	-3.04	-4.23	-4.23	-3.04	0.00	0.00
7	carry over		-3.38		-2.11		0.00		0.00	0.00	-7.27	-1.52	-2.43	0.00	-6.15	0.00	-1.52	0.00
8	balance	3.38	0.00	2.11	0.00	0.00	0.00	0.00	0.00	3.23	3.23	2.33	1.80	2.49	2.49	1.80	0.96	1.33
9	carry over		1.62		1.25		0.66		0.00	0.00	3.54	0.90	1.16	0.00	2.88	0.48	0.90	0.00
10	balance	-1.62	0.00	-1.25	0.00	-0.66	0.00	0.00	0.00	-1.63	-1.63	-1.17	-0.95	-1.32	-1.32	-0.95	-0.46	-0.64
11	carry over		-0.82		-0.66		-0.32		-0.14	0.00	-1.46	-0.47	-0.59	0.00	-1.29	-0.23	-0.47	0.00
12	balance	0.82	0.00	0.66	0.00	0.32	0.00	0.14	0.00	0.71	0.71	0.51	0.44	0.61	0.61	0.44	0.29	0.40
13	carry over		0.36		0.31		0.20		0.07	0.00	0.67	0.22	0.26	0.00	0.58	0.14	0.22	0.00
14	balance	-0.36	0.00	-0.31	0.00	-0.20	0.00	-0.07	0.00	-0.33	-0.33	-0.24	-0.21	-0.29	-0.29	-0.21	-0.13	-0.18
15	carry over		-0.16		-0.14		-0.09		-0.04	0.00	-0.29	-0.10	-0.12	0.00	-0.27	-0.07	-0.10	0.00
16	balance	0.16	0.00	0.14	0.00	0.09	0.00	0.04	0.00	0.14	0.14	0.10	0.09	0.13	0.13	0.09	0.07	0.09
17	carry over		0.07		0.07		0.05		0.02	0.00	0.13	0.05	0.05	0.00	0.12	0.03	0.05	0.00
18	balance	-0.07	0.00	-0.07	0.00	-0.05	0.00	-0.02	0.00	-0.07	-0.07	-0.05	-0.04	-0.06	-0.06	-0.04	-0.03	-0.04
19	carry over		-0.03		-0.03		-0.02		-0.01	0.00	-0.06	-0.02	-0.02	0.00	-0.05	-0.01	-0.02	0.00
20	balance	0.03	0.00	0.03	0.00	0.02	0.00	0.01	0.00	0.03	0.03	0.02	0.02	0.03	0.03	0.02	0.01	0.02
21	final	2.35	-2.35	1.32	-1.32	-0.48	0.48	0.10	-0.10	-4.67	8.98	-4.31	-3.58	-2.62	7.74	-1.54	-0.25	0.97
22																		
23		GK	GH	HG	HD	IE	IM	IJ	JI	JF	JN	JK	KJ	KG	MI	MN	NM	NJ
24	DF	0.291	0.209	0.419	0.581	0.368	0.368	0.265	0.209	0.291	0.291	0.209	0.419	0.581	0.581	0.419	0.419	0.581
25	FEM						-100				-100				-100			-100
26	balance	0.00	0.00	0.00	0.00	36.76	36.76	26.47	20.93	29.07	29.07	20.93	0.00	0.00	58.14	41.86	41.86	58.14
27	carry over	0.00	0.00	0.00	0.00	0.00	29.07	10.47	13.24	0.00	29.07	0.00	10.47	0.00	18.38	20.93	20.93	14.53
28	balance	0.00	0.00	0.00	0.00	-14.53	-14.53	-10.47	-8.85	-12.30	-12.30	-8.85	-4.38	-6.08	-22.86	-16.46	-14.85	-20.62
29	carry over	-3.04	0.00	0.00	0.00	-3.38	-11.43	-4.43	-5.23	-2.11	-10.31	-2.19	-4.43	0.00	-7.27	-7.42	-8.23	-6.15
30	balance	1.33	0.96	0.00	0.00	7.07	7.07	5.09	4.15	5.77	5.77	4.15	1.85	2.57	8.54	6.15	6.02	8.36
31	carry over	1.29	0.00	0.48	0.00	1.62	4.27	2.08	2.55	1.25	4.18	0.93	2.08	0.66	3.54	3.01	3.07	2.88
32	balance	-0.64	-0.46	-0.20	-0.28	-2.93	-2.93	-2.11	-1.86	-2.59	-2.59	-1.86	-1.15	-1.59	-3.81	-2.74	-2.49	-3.46
33	carry over	-0.80	-0.10	-0.23	0.00	-0.82	-1.90	-0.93	-1.05	-0.66	-1.73	-0.57	-0.93	-0.32	-1.46	-1.25	-1.37	-1.29
34	balance	0.40	0.29	0.10	0.13	1.34	1.34	0.97	0.84	1.17	1.17	0.84	0.52	0.73	1.58	1.13	1.11	1.55
35	carry over	0.36	0.05	0.14	0.00	0.36	0.79	0.42	0.48	0.31	0.77	0.26	0.42	0.20	0.67	0.56	0.57	0.58
36	balance	-0.18	-0.13	-0.06	-0.08	-0.58	-0.58	-0.41	-0.38	-0.53	-0.53	-0.38	-0.26	-0.36	-0.71	-0.51	-0.48	-0.67
37	carry over	-0.18	-0.03	-0.07	0.00	-0.16	-0.36	-0.19	-0.21	-0.14	-0.33	-0.13	-0.19	-0.09	-0.29	-0.24	-0.26	-0.27
38	balance	0.09	0.07	0.03	0.04	0.26	0.26	0.19	0.17	0.24	0.24	0.17	0.12	0.16	0.31	0.22	0.22	0.30
39	carry over	0.08	0.01	0.03	0.00	0.07	0.15	0.09	0.09	0.07	0.15	0.06	0.09	0.05	0.13	0.11	0.11	0.12
40	balance	-0.04	-0.03	-0.01	-0.02	-0.11	-0.11	-0.08	-0.08	-0.11	-0.11	-0.08	-0.05	-0.08	-0.14	-0.10	-0.10	-0.13
41	carry over	-0.04	-0.01	-0.01	0.00	-0.03	-0.07	-0.04	-0.04	-0.03	-0.07	-0.03	-0.04	-0.02	-0.06	-0.05	-0.05	-0.05
42	balance	0.02	0.01	0.01	0.01	0.05	0.05	0.04	0.03	0.05	0.05	0.03	0.02	0.03	0.06	0.04	0.04	0.06
43	final	-1.35	0.63	0.20	-0.20	24.99	-52.14	27.14	24.78	19.44	-57.50	13.28	4.14	-4.14	-45.25	45.25	46.12	-46.12

Table 5. Summary of final member end moment computations

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1								Member End Moments											
2		A	AE	B	BF	C	CG	D	DH	EA	EI	EF	FE	FB	FJ	FG	GF	GC	
3	final A	-3.46	3.46	-2.12	2.12	12.06	-12.06	-1.58	1.58	30.31	30.09	-60.39	89.29	4.25	-9.78	-83.75	62.27	-24.13	
4	final B	2.35	-2.35	1.32	-1.32	-0.48	0.48	0.10	-0.10	-4.67	8.98	-4.31	-3.58	-2.62	7.74	-1.54	-0.25	0.97	
5	final C	-16.49	16.49	-11.98	11.98	-9.64	9.64	2.02	-2.02	32.98	-65.36	32.38	29.13	23.97	-77.30	24.20	22.50	19.27	
6	final D	96.73	-96.73	98.94	-98.94	99.18	-99.18	71.10	-71.10	-93.47	88.00	5.47	3.87	-97.90	91.93	2.10	1.93	-98.37	
7	final F	57.97	-57.97	68.07	-68.07	82.31	-82.31	69.17	-69.17	0.52	-2.08	1.56	144.93	-43.06	-67.42	-34.46	111.52	-71.57	
8																			
9		GK	GH	HG	HD	IE	IM	IJ	JI	JF	JN	JK	KJ	KG	MI	MN	NM	NJ	
10	final A	-18.64	-19.50	-3.16	3.16	49.83	16.24	-66.07	68.99	-25.95	-23.53	-19.51	1.07	-1.07	9.70	-9.70	4.95	-4.95	
11	final B	-1.35	0.63	0.20	-0.20	24.99	-52.14	27.14	24.78	19.44	-57.50	13.28	4.14	-4.14	-45.25	45.25	46.12	-46.12	
12	final C	-54.20	12.42	4.03	-4.03	-80.18	78.70	1.48	-0.63	-90.56	73.91	17.28	37.30	-37.30	52.41	-52.41	-51.64	51.64	
13	final D	74.46	21.98	42.20	-42.20	66.16	-32.11	-34.05	-28.02	80.69	-18.44	-34.23	-46.46	46.46	-8.57	8.57	6.35	-6.35	
14	final F	-69.29	29.34	45.31	-45.31	11.80	4.50	-16.30	113.02	-88.66	-48.28	23.92	49.62	-49.62	-16.55	16.55	33.30	-33.30	
15																			
16								Member End Moments for Shear Calculations											
17		AE	EA	BF	FB	CG	GC	DH	HD	EI	IE	FJ	JF	GK	KG	IM	MI	JN	NJ
18	final A	-3.46	30.31	2.12	4.25	-12.06	-24.13	1.58	3.16	30.09	49.83	-9.78	-25.95	-18.64	-1.07	16.24	9.70	-23.53	-4.95
19	final B	2.35	-4.67	-1.32	-2.62	0.48	0.97	-0.10	-0.20	8.98	24.99	7.74	19.44	-1.35	-4.14	-52.14	-45.25	-57.50	-46.12
20	final C	-16.49	32.98	11.98	23.97	9.64	19.27	-2.02	-4.03	-65.36	-80.18	-77.30	-90.56	-54.20	-37.30	78.70	52.41	73.91	51.64
21	final D	96.73	-93.47	-98.94	-97.90	-99.18	-98.37	-71.10	-42.20	88.00	66.16	91.93	80.69	74.46	46.46	-32.11	-8.57	-18.44	-6.35
22																			
23									Shear Forces Calculations										
24	**shear**		EA		FB		GC		HD	EI	IE	FJ	JF	GK	KG	IM	MI	JN	NJ
25	Case A		-6.14		-0.53		3.02		-0.39	2.76	-10.56	-2.98	2.98	-1.64	1.64	-1.74	-6.06	-2.37	2.37
26	Case B		0.19		0.33		-0.12		0.03	2.83	-2.83	2.27	-2.27	-0.46	0.46	-8.12	8.12	-8.63	8.63
27	Case C		-1.37		-3.00		-2.41		0.50	-12.13	12.13	-13.99	13.99	-7.62	7.62	10.93	-10.93	10.46	-10.46
28	Case D		-0.27		16.40		16.46		9.44	12.85	-12.85	14.38	-14.38	10.08	-10.08	-3.39	3.39	-2.07	2.07
29																			
30																			
31			equilibrium at level							matrix equation for multipliers									
32	reactions		MN	IJK	EFGH			-16.75	21.39	-5.46	α		-7.59						
33	Case A		7.59	16.55	12.41			21.39	-55.13	42.76	β	=	-16.55						
34	Case B		-16.75	21.39	-5.07			-5.46	42.76	-95.46	γ		-12.98						
35	Case C		21.39	-55.13	40.02														
36	Case D		-5.46	42.76	-79.34														
37								α		-0.15	-0.08	-0.03		-7.59		2.88			
38								β	=	-0.08	-0.07	-0.03	*	-16.55	=	2.14			
39								γ		-0.03	-0.03	-0.02		-12.99		0.93			

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