Factors Affecting Students' Ability to Estimate Angles

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Abstract

Since the establishment of the mathematics standards by the National Committee for Teaching Mathematics (NCTM), researchers have been evaluating these standards on successful implementation strategies and student achievement. One area within these standards focuses on competencies students need to master in basic geometry. The NCTM standards describe geometry as a way to provide students the ability to visualize and work with spatial relationships and estimation. The authors assessed these standards that relate to geometry and looked into the ability of students to estimate angles.

During the 1999-2000 academic year, a study was conducted to determine the competency of students taking engineering graphics courses when estimating both simple and complex angles. A mathematician originally developed the instrument used within the study in the 1960's (Maletsky, 1966). The study consisted of 262 students (mainly engineering majors in their sophomore year) from different academic disciplines from across campus taking an introductory or advanced engineering graphics class. All students were asked a series of questions that related to visualization experience and were asked to either estimate an angle using no instrumentation, or construct, to the best of their ability, given angles using only a straightedge and pencil.

Major conclusions centered on the fact that there were no statistically significant correlations between majors and experiences in and outside of the classroom for any group when asked to estimate an angle, except those students required to estimate or construct an obtuse angle if taking their first engineering graphics course. The authors will explain their analysis of the findings and show other influences in student's decision-making process for estimating both simple and complex (dihedral) angles.

Introduction

From an early beginning, humans have used graphical representations to communicate ideas. Engineers and other professionals related to science, mathematics and technology have long used geometry and descriptive geometry to find solutions to everyday problems. In fact, geometry can be defined as a science to use graphic representations to find solutions to spatial problems (Pare, E. G., et al., 1997). These spatial problems require the ability to use spatial visualization to mentally manipulate and interpret visual information in problem solving situations (Wiley, 1990). Although geometry and spatial visualization play an important role in everyday activities, Perkins (1982) conducted studies that indicate humans are basically poor geometers. The rationale for such a statement came through a series of studies analyzing geometric factors like rectilinearity, symmetry, and extrusion (both linear and curved). Through these research studies, Perkins concluded that the human perceiver does act as a geometer, but a "sloppy" one, and more training is needed to associate geometry to real-world examples so that humans can use geometry accurately and in everyday situations.

Geometry and descriptive geometry are not the only areas requiring student skill development. Estimation plays an important role in everyday life as well. The National Council for Teachers of Mathematics (NCTM) defines estimation as a process involving comprehending a problem, relating the information to data known, making judgements, and verifying reasonableness. Estimation is seen as a process to connect mathematical ideas to the physical world and communicate these ideas through articulation. Harte and Glover (2000) stated that many situations involve estimation rather than precision and that teachers need to help students develop good estimation skills. Happs and Mansfield (1992) argue that learning to estimate can be difficult, but students with a prior or contemporaneous experiences in measurement (ie. geometry), find it less difficult to apply these estimation skills. Students use of mental imagery, as learned through geometry, will "benefit from opportunities to construct an image in the same way that scientists and engineers construct mental models to serve as useful representations of the phenomena to be understood" (Happs & Mansfield, 1992, p. 46). But, if students are to develop these skills in estimation, direct linkages to geometry and its use in everyday life must be taught in both elementary and secondary schools.

Geometry and estimation go "hand-in-hand" for the engineer or technologist. This study was designed to check if engineering students could apply their knowledge of geometry and other visual skill development classes to problem-solving situations (ie. estimating angles) and to look into students' backgrounds to see if any correlations exist that may have been influential in their angle estimation ability.

Justification of Research

In 1990, the National Assessment of Educational Progress (NAEP) conducted a series of statewide assessments in mathematics. The assessment focused on both fourth and eighth-grade students in public institutions. North Carolina participated in this voluntary state-by-state assessment with testing in areas of numbers, data analysis, geometry, basic mathematics, algebra, and estimation. North Carolina students performed lower than the national average in all six areas, particularly in geometry and estimation. Once this information was known, the NAEP asked teachers about the amount of time spent on each of the six areas that were assessed. Only twelve percent of North Carolina teachers indicated they place an emphasis in Geometry at the fourth-grade level, and only fourteen percent do so at the eighth-grade level. As for estimation, no teachers in North Carolina place emphasis in this area (National Center for Education Statistics {NCES}, 1992). Also, in 1986 the Massachusetts State Department of Public Instruction (1987) reported that geometry was a subject that was "highly ignored", based on its academic utility for visualization skills, logic, and measurement (estimation) abilities. In this report, geometry is credited with the development of visual and spatial skills as well as providing a way for the understanding and ordering of the immediate surroundings, through the avenue of problem solving. The report also stated that fewer than half of students were able to calculate the degree measurements of an angle, and only fory-five percent were able to estimate length with some accuracy. Only fifteen percent of seventh-graders and thirty-six percent of eleventh-graders were able to give the area of a right triangle, given the measurements for each leg. Overall, the study concluded that more attention needed to be given to this important area so students could easily apply geometry to problem-solving situations, the work environment, and everyday activities. Support for these conclusions came from the NCTM in 1989 when the committee recommended adopting a new series for mathematics instruction that highlighted probability, statistics, mental mathematics, estimation, and geometry (Vann, 1995).

Geometry and descriptive geometry have been a part of the engineering curriculum since the 1930's. Of all college subjects, descriptive geometry has been reported to be the only subject that both develops and utilizes visual ability. Engineering graphics researchers through the years have determined that outside influences in early child development also develop students' ability to visualize (Miller, 1996). Deno (1995), indicated that engineering students coming from a traditional curriculum in the American education system, are not always equipped with good visual skills. In Deno's research, it was determined that the type of toys engineering students had in childhood could play an important role in the development of their visual capabilities, thus improving students skill ability in areas of geometry and estimation. Hilton (1985) also stated those early childhood experiences with toys, games, and tools do affect students' ability to visualize. In his studies, he found gender differences based upon culture influences. In societies where females are subjugated, their visual abilities were poor, but in cultures where both sexes are allowed independent lifestyles, both sexes had good visual skills. These studies led the researchers for this study to ask questions to students about early childhood experiences and to compare this information to their angle estimation skills to see if a direct link between these two areas would exist within the studied population.

Several researchers have indicated a need for further studies in areas associated with visual abilities, geometry, and graphic representation. Waters (1984) stated that among the topics of both formal and informal geometry are measurements of angles and construction of angles. In his research, he indicated that students are taught to measure and construct angles using a protractor and ruler, but seldom are taught how to estimate an angle. Waters also stated that in the real world, estimating angles is as frequent as accurate measuring. He conducted a study with post-secondary students to see if they could estimate angles accurately. Overall, his results stated that older students were better at estimation and that students had a tendency to over estimate. He concluded the study by stating that more research was needed to determine the reason why students seem to have difficulty estimating angles. Wilson and Davis (1985) stated that more research is needed in areas associated with the development of visual skills. In their research, they determined that if students have difficulty in geometry and/or visualization, it might represent a potential difficulty in graphic conceptualization, and perhaps affect academic success. Therefore, they determined that in order to understand how students' development and use of visual skills, more research is needed at the national level. Frazier (1988) determined that one could determine abilities to visualize by asking students to estimate and construct angles associated with solving geometry problems. After conducting a series of angle estimation studies, Frazier determined that no conclusive evidence existed to link visual abilities to successful problem solving. Therefore, more research is needed to see if students can apply angle estimation (i.e. visual abilities) to problem solving situations.

Engineering graphics educators have always considered estimating skills as being an important component in their curricula. Most, if not all, courses taught as engineering/technical graphics have some form of descriptive geometry, angle development, and problem solving as a part of their subject matter (Werner and Claderon, 1985; Hotchkiss and Moore, 1982). Considering these components, the researchers for this study wanted to link the above research findings and methodologies and develop a study that would analyze college students' abilities to estimate angles, the problem solving skills they use to do so, and compare this information to their childhood backgrounds, training and experiences to see if students really do use these skills and abilities to estimate angles accurately. Since some students have more extensive backgrounds in graphics and geometry, as compared to others in the study, students in advanced courses were also given more complex angles (dihedral angles) to estimate. As previous research has indicated, few attempts have been made to link these areas and even fewer studies have researched students' abilities to estimate and understand complex angles like dihedrals (Chin, 1990).

Methodology

The research study used a validated survey test for collecting the data. This test used for assessing student's abilities to estimate angles came from Waters (1984), who used it with college students in the mid-1980s. Maletsky (1966) originally developed the test for use with middle school students in New Jersey. Later, Protomastro (1979) made improvements to the test. Also, for this study the researchers included additional questions about student backgrounds. In addition to estimating two-dimensional angles, advanced students estimated dihedral angles using three models.

The population that participated in the study were students taking a Graphic Communications course (i.e. engineering graphics) at NC State University during the spring semester 2000. Two groups of participants were included in the research. The first group consisted of students taking introductory level courses in Graphic Communications. This group, although including diverse majors, consisted primarily of engineering students. Students were stratified by the different introductory sections of classes offered. One-half constructed angles with just a pencil and straight edge, while the other half were asked to look at given angles and estimate their value. The second group included students in advanced Graphic Communications classes who were taking additional graphics related classes as a minor or major in Graphic Communications, or just as an elective. These students had at least one engineering graphics related course before entering these advanced classes. These students took the same test given to the introductory level participants with additional estimations made for dihedral angles. These students were asked the same questions as the introductory classes, but were asked to estimate given angles. The test was given during the first week of class of the spring semester 2000, and administered by the researchers for consistency.

Findings

The first group of participants, those who were taking introductory level graphic communications course, had a total of 262 participants. Most participants were male, between the ages of 19 and 20, with the classification of sophomore. The majority was right-handed, majoring in either aerospace, mechanical, or civil engineering. Table 1 shows the major demographic areas for this introductory group.

Table 1
Demographics for Participants in Introductory Level Courses n=262

GROUP	Frequency	Percent%
Constructed Angle	112	42.7
Estimated Angle	150	57.3
GENDER		
Male	208	79.4
Female	54	20.6
AGE		
Under 19	13	5
19-20	157	59.9
21-22	55	21
Over 22	36	13.7
CLASSIFICATION		
Freshman	35	13.4
Sophomore	136	51.9
Junior	58	22.1
Senior	28	10.7
Other	5	1.9
HANDEDNESS		
Right	230	87.8
Left	32	12.2
TOP 5 MAJORS		
Mechanical	64	24.4
Civil	60	22.9
Aerospace	27	10.3
Industrial	25	9.5
Other	86	32.9

Forty-two percent (110) of the introductory graphics group indicated this was their first graphics course and 77.9% of this group said the introductory engineering graphics course they were in was required for their major. Only 25.2% (66) of the 262 in this group had work experience related to any form of graphics and most (91.2%, n=239) indicated they learn best through pictures. Seventy-three percent, or 193, had computer training and only 35.1% (92) had any previous computer-aided design (CAD) courses. For those who had CAD experience, AutoCAD was the most used (30.9%, n=81) and only 10.3% (27) had any modeling experience. Two hundred and fifty-nine participants (98.8%) indicated they had taken geometry in high school.

Based upon Deno's (1995) research regarding activities that correlate with visual skill development, students were asked about their level of participation in some common early childhood experiences. Table 2 shows the most recognized types of toys participants played with as children.

Table 2
Top Five Early Childhood Experiences (Toys) Related to Visualization n=262

Types	Frequency	Percent%
Lincoln Logs	182	69.5
Rubics Cube	197	75.2
Etch-a-Sketch	222	84.7
Puzzles	235	89.7
LEGOS	241	92

*NOTE: Total percentage for each category is 100.

The participants in the introductory graphics group were asked to either construct or estimate given angles. The first question, utilizing an angle of 36°, had a combined (both constructed and estimated) mean of 39.42 with a standard deviation of 11.61. The second angle required (56°) had a combined mean of 55.37 with a standard deviation of 9.01. The third and final angle estimate for the introductory graphics group was 144°, with a group mean of 137.59 and a standard deviation of 14.68.

The researchers analyzed the data by comparing the two sets within the group; one set constructed the given angles (n=112), the other gave estimates of given angles (n=150). T-tests were used to determine if one group was statistically different from the other using an alpha of $p<.05$. The question involving the 36° angle showed no statistical difference between those participants asked to construct the angle as compare to those that estimated the given angle (t=.36). The second question using an angle of 54° yielded a statistical difference with a t=.02. Those students asked to construct the angle had a mean of 53.84 (SD=10.81). Participants who gave the estimate had a mean of 56.51 (SD=7.22) for the given angle. The question involving the 144° angle had statistical difference as well, with a t=.0001. Participants asked to construct the third angle had a mean of 133.38 (SD=13.51). Those that estimated the given angle had a mean of 143.25 (SD=14.33).

The researchers also compared the two sets (those who estimated the angles and those who constructed the angles) with other variables to see if any statistical significance could be found relating angle estimations to demographic information. Below are those comparisons that were found significant using t-test for the two sets combined together and individual sets, with an Alpha of $p.<.05$.

In the set of data for those that were asked to estimate a given angle only, gender was found to be statistically different for the first angle (36°) with a t=.02. Males had a mean of 39.23 (SD=8.92, n=124) and females had a mean of 43.76 (SD=9.34, n=26). The second angle (54°) was also found to be statistically different for this group that gave an estimated angle only, with a t=.05. Males had a mean of 57.19 (SD=6.52) and females had a mean of 53.26 (SD=9.37). The problem using the 144° angle had no statistical difference for this group.

For the entire group of students in introductory graphic communications courses (i.e. engineering graphics), there was a significantly different estimate (t=.0008) of the first angle (36°) between students indicating that this was their first graphics related course and those with a previous graphics course. Those participants that indicated this was their first graphics course had a mean of 42.43 (SD=13.92, n=110) as compared to those who had a previous graphics course (M=37.25, SD=9.03, n=152). Considering the students who constructed the angle and those who estimated the angle individually, students who constructed the first angle (36°) had statistical difference (t=.03) between those who indicated this was their first graphics course (M=42.47, SD=19.18, n=46) and those students who had previous courses related to graphics (M=35.95, SD=8.71, n=66). The same held true for students asked to estimate the given angle. A statistical difference (t=.004) was found between those students who indicated this was their first graphics course (M=42.40, SD=8.53, n=64) as compared to those that had previous courses related to graphics (M=38.24, SD-9.20, n=86). No other statistical difference was found for the other two angle estimates, either considering the group as a whole or as two groups based on method of estimation.

When comparing the variables of estimating angles to whether or not students had a previous CAD course, only those who estimated (not constructed) the third angle (144°) had any statistical difference (t=.04). Participants who had a previous CAD course had a mean of 135.93 (SD=9.71, n=59). Those who did not have any CAD experience had a group mean of 131.72 (SD=15.31, n=91). No other statistical significance was found between these variables both as one group or broken-out into the two different sets of data.

An ANOVA was performed on the total group of introductory graphics students to see if any one major was statistically different from the others in accuracy of estimating the three angles. Only the second angle (54°) was found to be statistically significant with an F value of 1.86, p.=.02 for the entire group. When divided into the two sets (constructed and estimated), the ANOVA test found statistical significance between majors who constructed the second angle (F value=7.27, p.=.0001) only. Since no other significance was found for the other two angles using the ANOVA test both as one group and in the individual sets, and the population for each major varied greatly between the participants, this test had limited ramifications on the results found within the study.

The advanced students that participated in the study, those having taken at least one or more graphics related course, were asked to estimate only (not construct) the same angles the introductory graphics group estimated. Also, this group (n=58) was given three models with dihedral angles and were asked to estimate those angles. It was hypothesized by the researchers that advanced graphics students, given their experiences, should estimate angles more accurately than students in introductory graphics courses. Also, these advanced students could, given their previous classes, estimate more complex angles like dihedrals. The demographics for this advanced group are shown in Table 3.

Table 3
Demographics for Participants in Advanced Level Courses n=58

GENDER	Frequency	Percent%
Male	52	89.7
Female	6	10.3
AGE		
19-20	23	39.7
21-22	21	36.2
Over 22	14	24.1
CLASSIFICATION		
Sophomore	20	34.5
Junior	12	20.7
Senior	25	43.1
Other	1	1.7
HANDEDNESS		
Right	51	87.9
Left	7	12.1
TOP 2 MAJORS*		
Mechanical	22	37.9
Education	19	32.8

NOTE: Maximum percentage for each category is 100.

Thirty-seven percent (22) of these advanced students had work experience related to graphics and 93.1% (54) said they learn best through pictures. Over 70% (41) indicated that they had some form of computer training and that AutoCAD (91.4%, n=53) was their CAD software of choice. Sixty-two percent (36) had used modeling software with 3D StudioMax and XRES being the most used by these students. As for early childhood experiences that related to Deno's (1995) work with developing visual skills, Table 4 shows the results for the advanced graphics students. Table 5 shows the mean estimated angle for the group for both types of angles, regular and dihedral.

Table 4
Top Five Early Childhood Experiences (Toys) Related to Visualization for Advanced Graphics Students n=58

Types	Frequency	Percent%
Lincoln Logs	43	74.1
LEGOS	53	91.4
Puzzles	51	87.9
Etch-a-Sketch	48	82.8
Rubics Cube	49	84.5

*NOTE: Total percentage for each category is 100.

Table 5

Average Degrees for Advanced Graphics Students for Angles n=58

Angle/True Angle	Mean	SD
AngleA/36°	36.86	12.35
AngleB/54°	57.27	5.14
AngleC/144°	137.31	20.17
Dihedral A/62.8°	55.51	10.58
Dihedral B/94.7°	87.84	10.34
Dihedral C/122.7°	119.74	19.52

The researchers did the same comparisons as indicated above with the introductory graphics group with the following outcome. Statistical difference (t=.04) was seen for the second dihedral angle (94.7°) when comparing students estimates for this angle to work experience. Those students with work experience had a mean of 90.68 (SD=2.33, n=22), those without work experience had a mean of 86.11 (SD=12.76, n=36). No statistical difference was found in other variables, including major and gender, for the advanced graphics group.

The final analysis of data was to compare those students who estimated (not constructed) the three angles between the introductory and advanced students. The introductory graphics group (n=150) who estimated these three angles were compared with the entire advanced graphics group (n=58) that only estimated these angles. Using an ANOVA test to compare these groups, statistical difference was found between the two groups, with an F value of 4.07, p.=.04, and the accepted Alpha of p>.05.

Conclusions

The researchers found very little evidence that either supported the assumption that students could estimate or could not estimate angles. But, given the data collected, some trends in the data are worth discussing. First, the students in graphics classes are largely engineering majors. Second, looking at early childhood experiences, no conclusive evidence was shown that these experiences develop visual and spatial relations abilities in students, but one must note that for both introductory and advanced groups, the same five toys were identified as being the most common for participants in both groups. Third, it seems that gender could play a role in a student's ability to estimate angles. Males seem to estimate angles better than females, but no conclusive evidence was found as to why, including early childhood experiences. Fourth, when comparing students in their first graphics course to those that had some type of previous training in graphics, a trend is shown that those with a graphics background do estimate angles better than those that had little to no previous graphics related training (i.e. courses). Considering this, students who had a CAD course also estimated angles better than those students with no CAD background, and students in advanced courses did better at estimating the common three angles. This conclusion supports the need to provide students with this type of training early in their academic careers. Next, for students with advanced training in graphics, it was still difficult for students to estimate complex angles (i.e. dihedrals). As indicated by the large standard deviations and the group means, students' estimates for these angles were based upon one plane, not two. Therefore, this supports the need to include more descriptive geometry in the engineering curriculum so students can deal with complex planes and surfaces.

Overall, given the data collected and analyzed in this study, students are still making the same mistakes as reported in other studies similar to this one (Waters, 1984; Protomastro, 1979; Maletsky, 1966). Students have the tendency to over estimate acute angles, and under estimate obtuse angles. Reasons for this are not known, nor could they be delineated from this study. But, one can conclude that estimation is an important skill that needs practice and constant improvement. More research in student use the skills taught in geometry and graphics classes is needed so that professionals, in the field of engineering/technical graphics, can improve the curricula offerings to students and in the long run, better engineering and technology programs.

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