Electric Circuit Analysis Using Voltage Maps and PSpice®{ TC \l1 "}

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Abstract

Engineering students as a basic course in most curricula study electric circuit analysis. It is taught using procedures developed over 60 years ago in textbooks written by the staff at MIT. Those procedures have been followed by other textbook authors, continuing the use of confusing symbols for current and potential difference that plague beginning students of electric circuit analysis.

This paper proposes the use of a closed-head arrow for current direction, and an open-head arrow for a voltage rise between two nodes. In this method of identification all voltages are positive and negative rises only. All circuit voltage rises can be drawn in a *voltage map*, using open-head arrows to show both source potentials and Ohm's law voltages across other elements. The beginning student can use it to quickly learn circuit behavior in terms of voltage and current locations that are related to Ohm's law and Kirchhoff's circuit laws. The proposed method correlates with terminology used in the computer simulation program called PSpice®[1]. The voltage map shows at a glance that there is only one potential difference across every parallel path between any two nodes. A voltage rise between two nodes equals the algebraic sum of voltage rises in every other parallel path from one node to the other. It is a clearer picture of Kirchhoff's voltage law (KVL) than a narrative statement. The voltage map is an enhancement to learning circuit analysis.

Introduction{ TC \l3 "}

Engineering students learn to analyze electric circuits using three basic circuit laws and their interactions in circuit operation. They are Ohm's law, Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL). Other circuit principles are merely extensions of these three, including parallel current division, series voltage division, Thévenin's theorem, Norton's theorem, loop current analysis, and node voltage analysis. How to use these tools and interpret results from them requires understanding basic concepts found in these three circuit laws.

The pioneering work over 60 years ago by MIT faculty who wrote textbooks for study of the electrical sciences set the pattern for electric circuit analysis in the United States. Authors who developed textbooks after those early works largely followed the procedures advanced there. This paper extends the analytical procedures adopted for identifying current direction and voltage polarities in a circuit. While the MIT works adopted an arrow symbol to indicate current direction, a similar arrow symbol was not used for voltage. Instead, double subscript notation was adopted, together with plus (+) and minus (-) signs for relative polarity of node voltages. This notation has contributed to confusion for students who are learning electric circuit analysis. A simpler open-head arrow for internodal voltage polarity promotes a better understanding of electric circuit operation.

College courses in circuit analysis currently depend on textbooks that follow patterns of analysis laid out by the pioneering works written at MIT in the early 1940's. "*Electric Circuits*" by the electrical

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[1] PSpice is a registered trademark of OrCAD, Inc., 9300 SW Nimbus Ave., Beaverton, OR 97008 USA

engineering faculty and "*The Mathematics of Circuit Analysis*" by E. A. Guillemin are classics in this field[2]. Their published procedures for writing circuit equations are ingrained into the study of electric circuits. However, the current and voltage symbols used in those early works often obscure the simplicity of voltage distribution imposed by Ohm's law and Kirchhoff's circuit laws.

The Voltage Map of a Circuit{ TC \l3 "}

The beginning student of electric circuit analysis may be confused by early terminology and symbols used to denote voltage polarity in a circuit. This paper develops a consistent conformity between a circuit schematic diagram and internodal voltage distribution in the circuit. It is useful for analysis of dc, ac, and combined circuits, with active as well as passive components. The MIT book adopted the practice of using plus (+) and minus (-) signs and double-subscript notation for potential differences. Later authors followed that practice in textbooks written for the study of electric circuits.

This paper proposes a closed-head arrow to show current placement and direction in a branch. It also proposes an open-head arrow to denote an increasing potential between nodes. An open-head arrow shows increasing voltage, a *voltage rise*, in the direction of the arrow between nodes. All internodal voltages are shown as voltage rises, whether they are potential sources or potential differences across circuit elements. By using open-head arrows for internodal voltage rises across all circuit components, a *voltage map* of potential differences can be drawn in the same topology as elements in the circuit schematic diagram. These two arrow symbols remove ambiguity about locations of current and voltage polarities that may result from plus (+) and minus (-) signs and double-subscript notation found in other works. The voltage map is related to KVL. It can be used as an enhancement to learning to achieve a better understanding of circuit operation for the neophyte student. Voltage map analysis can be applied to complex series and parallel combinations of circuit components, and is easily related to output data from PSpice simulation of circuit operation. It can be applied to analyze circuits containing sources of dc, ac, and their combinations, in active as well as passive circuits.

Drawing the Voltage Map of a Circuit{ TC \l4 "}

The closed-head arrow in Figure 1 defines the direction of conventional current I in the resistor R. The closed head of the arrow suggests an enclosure of moving charge, indicating movement of charge inside the resistor in the direction of the arrow. This is *conventional current*.

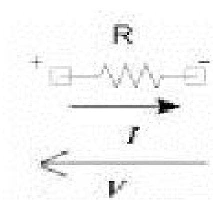


Figure 1. Current and voltage rise arrows express Ohm's law, $V = IR$.

In Figure 1 the open-head arrow points in the direction of increasing potential between the negative node and the positive node of the resistor. It is placed between nodes, indicating the internodal

[2] See References.

voltage rise V across resistor R. This is the voltage rise across R due to current I in the opposite direction. Figure 1 expresses Ohm's law for a resistor in terms of two arrows, $V = IR$. The potential difference across a resistor equals the product of resistance and the current in the resistor. Its polarity direction (the voltage rise) is opposite of the current direction.

In a similar way a decrease in potential from one node to another is a *voltage drop*, from "+" to "-". *A voltage drop, then, is the opposite of a voltage rise.* It can represent a loss of electrical energy from its conversion to another form such as heat. A **{** XE "Voltage:voltage drop" **}{** XE "voltage drop" **}**voltage drop to the right across R has the same magnitude potential difference as the voltage rise in the opposite direction. The open-head arrow is used for voltage rises only. This polarity is consistent with PSpice notation. Voltage across a component has current into the positive node. A separate symbol for voltage drop is neither necessary nor desirable. A voltage drop is simply a negative voltage rise. A mathematical minus sign is used to relate a voltage drop as opposite of a voltage rise between the same two nodes. For resistor R there is a negative voltage rise $-V$ from left to right, in a direction opposite of the open-head arrow for voltage rise. Every positive voltage rise between two nodes is also a negative rise in the opposite direction.

Both current direction and voltage polarity can be shown clearly using only two arrow symbols. These two arrows show current direction in the circuit and the direction of increasing voltage between nodes. The closed and open arrowheads distinguish current from voltage rise.

Circuit Analysis of a Voltage Map{ TC \l4 "}

Figure 2 depicts the voltage map of a simple dc circuit connected in series-parallel. Note the use of arrows for assumed positive branch currents and component voltage rises. Current arrows show KCL at every node, and voltage rise arrows show KVL for every closed loop. A voltage rise arrow also indicates open-circuit voltage between nodes where more than one circuit element is present. Analysis will determine whether assumed directions are valid. Calculated negative values of current and voltage rises show that actual positive directions are opposite of those assumed.

In the schematic in Figure 2(a), currents I_1 and I_2 show KCL at the top node, with currents into the node equal to currents out of the node. There is no current in source V_3 because of the open circuit. Each resistor has a voltage rise across it related to current in it, in accordance with Ohm's law. In (b) there are voltage rise arrows for sources and resistors that make up a voltage map of the circuit. All voltages are shown as rises in the direction of assumed increasing potential. These assumed voltage rises are correct from basic concepts for sources and Ohm's law polarity for resistors. The only unknown voltage rise is V_{oc}. Its actual polarity can be found from KVL conditions in the voltage map.

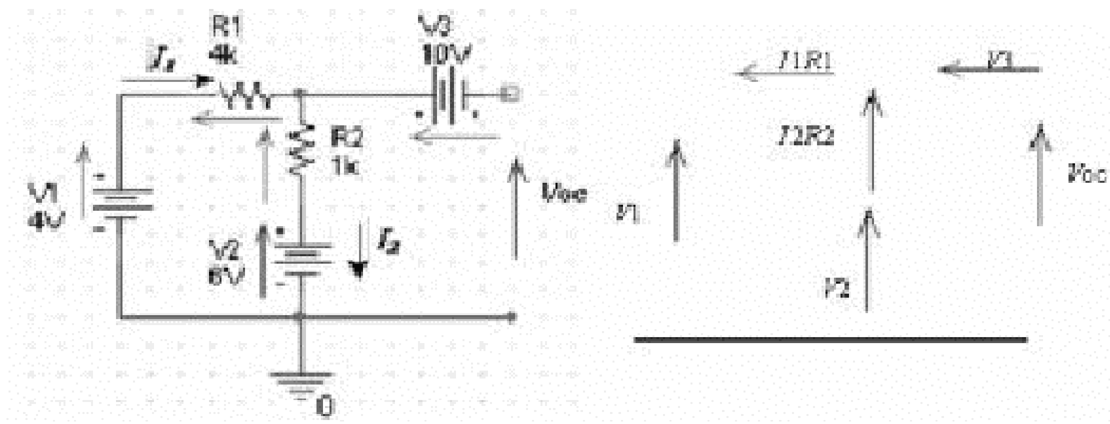
The usual statement of KVL includes all voltages in a closed loop, stating the algebraic sum of voltage rises in the same direction around a closed loop of circuit components equals zero. This KVL statement may be paraphrased.

Corollary to { XE "voltage loop:KVL for a closed voltage loop" }KVL There is only one potential difference across every parallel path between any two nodes. A voltage rise between any two circuit nodes equals the algebraic sum of voltage rises in any other parallel path between them.

For example, in Figure 2(b) the voltage rise V_{oc} at the right is equal to the algebraic sum of voltage rises in the middle branch between the same two nodes,

$$V_{oc} = V_2 + I_2R_2 - V_3$$

In this circuit $I_2 = (V_1 - V_2)/(R_1 + R_2) = (4V - 6V)/(4k\Omega + 1k\Omega) = -0.4$ mA. Then the open-circuit voltage $V_{oc} = 6V - 0.4V - 10V = -4.4V$. This negative result means that V_{oc} is a positive rise downward between the open-circuit nodes, rather than its assumed upward direction in the voltage map.



(a) Schematic of the circuit (b) voltage map of the circuit

Figure 2. Series-parallel dc circuit with current and voltage rise arrows.

Similarly, KVL shows that V_{oc} is equal to the algebraic sum of voltage rises included in the parallel path of the left branch,

$$\begin{aligned} V_{oc} &= V_1 - I_1R_1 - V_3. \\ &= 4V + 1.6V - 10V \\ &= -4.4V \end{aligned}$$

The same voltage rise is found across these nodes in terms of any parallel path between them (KVL).

The alternative statement of KVL is easily learned and applied by beginning students. It avoids having to determine whether a potential difference is a voltage rise or drop, because only arrow directions are involved in writing circuit equations. Potential differences in the arrow direction are positive voltage rises; those in the opposite direction are negative rises.

Comparison with PSpice Results{ TC \l4 "}

Voltage map analysis can be compared with results from PSpice simulation of the circuit. The circuit schematic is set up for PSpice analysis in Figure 3. Resistor R_{open} provides a component to develop V_{oc} in PSpice. PSpice cannot evaluate open circuits, so a large value resistor, 1E20Ω, is connected to develop open-circuit voltage.

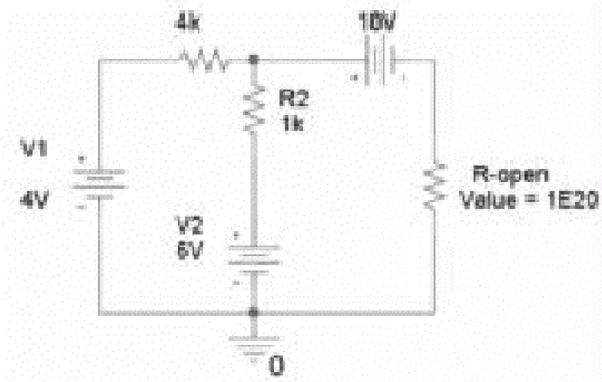


Figure 3. Schematic layout for PSpice simulation of the circuit.

Results from PSpice analysis can be readily interpreted because of similar notations for current directions and voltage polarities. In PSpice, current in a component is assumed positive into the positive node, whether it is a source or other element. Each node voltage is listed as a rise from node 0 to the numbered node. In the output file from PSpice simulation listed below, node N00082 is the negative node of source V_3. Node voltage at N00082 is listed as -4.4000V. Open-circuit voltage V_{oc} occurs across R_{open} as a negative voltage rise from 0 to node N00082. This is the same as calculated above, where the rise V_{oc} = -4.4V.

Source currents listed in the output file also show a correct calculation of current in V_2. In the original circuit, current I_2 was assumed positive into source V_2, but the output file shows it is -4.000E-04 = -0.4 mA. This means that actual current in source V_2 is out of its positive node, which was calculated above. Current in source V_1 is listed as 4.000E-04. This result means that positive current of 0.4mA is into the positive node of V_1. This is correct because KCL requires $I_1 = I_2$. Since I_2 is counterclockwise, I_1 must be down into the positive node of V_1, therefore positive in PSpice notation.

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****     CIRCUIT DESCRIPTION
**** INCLUDING aseefig3-SCHEMATIC1.net ****
* source ASEEFIG3
V_V3        N00032 N00082 10V
V_V2        N00014 0 6V
V_V1        N00018 0 4V
R_R2        N00014 N00032  1k
R_R1        N00018 N00032  4k
R_R-open        0 N00082  1E20

 ****    SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000  DEG C
**********************************************************************
 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE VOLTAGE
(N00014)    6.0000       (N00018)    4.0000       (N00032)    5.6000       (N00082)   -4.4000

   VOLTAGE SOURCE CURRENTS
   NAME        CURRENT
   V_V2       -4.000E-04
   V_V1        4.000E-04
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Analytical techniques developed in this paper for a simple dc circuit can be extended to complex series-parallel circuits, using dc, ac, and combined voltage and current sources. Where ac sources are used, phasor notation is used for peak values and phase angles to represent voltage rises at some point in time. Analysis proceeds in any circuit configuration from fundamental applications of Ohm's law and Kirchhoff's laws. Calculations can be compared with results from PSpice simulation, with special attention to polarity conventions required to interpret numerical values.

Summary{ TC \l5 "}

This paper proposes the use of a closed-head arrow for current direction, and an open-head arrow for a voltage rise between two nodes. In this method of identification all voltages are rises only. All circuit voltages can be drawn in a *voltage map*, using open-head arrows to show both source potentials and Ohm's law voltage across other elements. The voltage map shows at a glance that there is only one potential difference across every parallel path between any two nodes. A voltage rise across any branch equals the algebraic sum of voltage rises in any other parallel path between its two nodes. A voltage map is a clearer picture of KVL than a narrative statement. The proposed voltage map method of circuit analysis correlates with terminology in the computer simulation program called PSpice.

The voltage map technique for circuit analysis is valid for dc, ac, and combination modes in a circuit. It is consistent with circuit laws and requires no new techniques to implement. Ohm's law and Kirchhoff's circuit laws can be taught from the advantage of arrow symbols for both current and voltage. By showing distribution of voltages in a circuit, the voltage map enhances learning by providing a bridge between circuit operation and its computer simulation. It has consistent conformity with circuit component layout in the schematic diagram of the circuit. The voltage map procedure is easily learned by students of circuit analysis. The beginning student can use it to quickly learn circuit behavior in terms of voltage and current locations that are related to Ohm's law and Kirchhoff's circuit laws. The voltage map shows at a glance that there is only one potential difference across every parallel path between any two nodes. A voltage rise across any branch equals the algebraic sum of voltage rises in any other parallel path between its two nodes. The voltage map is an enhancement to learning circuit analysis. It is a clearer picture of KVL than a narrative statement. The voltage map is an innovative extension of the analytical methods developed for circuit analysis over 60 years ago.

The usefulness of the voltage map procedure in electric circuit analysis can be applied to other physical systems as well. Fluid flow, heat transfer, mass transfer, mechanical vibrations, and other cases of energy conservation, can be analyzed in a similar way. The voltage map concept uses analogous variables in those systems, substituting for current and voltage in electric circuits.

References{ TC \l3 "}

1. Faculty of the Electrical Engineering Department (1940) "*Electric Circuits*," The Massachusetts Institute of Technology, Cambridge, MA.

2. E. A. Guillemin (1949) "*The Mathematics of Circuit Analysis*," The Massachusetts Institute of Technology, Cambridge, MA.

Russell E. Puckett{ TC \l4 "}

Russell E. Puckett, PE, is Professor Emeritus at Texas A&M University. He retired in 1992 after 35 years teaching electrical engineering and engineering technology. During that period he taught electric circuit analysis using many different textbooks. Because of confusing analytical methods used by different authors to analyze electric circuits, Professor Puckett developed and used the voltage map procedures presented in this paper to help students learn a simpler approach to understanding circuit operation.

In addition to his teaching assignments, he was active in many other related academic assignments. Some of them are listed below.

- Director of student projects for practical applications of technology.
- TAC/ABET evaluator of baccalaureate engineering technology programs for accreditation at public colleges and universities.
- Manuscript editor-reviewer for publishers of electrical and electronic circuits textbooks.
- Technical writer consultant for international oil exploration corporation.
- Field engineer and design engineer for industrial and federal organizations.
- Administrator of engineering research programs and projects for College of Engineering.
- Administrator of student cooperative education programs for College of Engineering.

Honors and Awards

- Outstanding Teaching Award, Former Students Assn., Texas A&M University, 1992
- Professor Emeritus, Texas A&M University, 1992
- Outstanding Faculty Award, Texas A&M University, 1985, 1989
- Outstanding Student Branch Counselor Award, IEEE Region 5
- (one of ten annual international awards), 1985
- Science Faculty Fellowship Award, National Science Foundation, 1965
- Awarded U.S. patent 3,220,115 for Sine Wave Template, 1965
- Listed in *Who's Who in Engineering, American Men and Women in Science,*
- *Who's Who in the South and Southwest*
- Member, Tau Beta Pi (Engineering Honor Society),
- Eta Kappa Nu (Electrical Engineering Honor Society),
- Tau Alpha Pi (Engineering Technology Honor Society)

Author of textbooks

- "*Introduction to Electronics*", John Wiley & Sons, 1968
- "*Laboratory Manual for Introduction to Electronics*", Buck Engineering Co., 1969
- "*Introduction to Electronics*", (2nd Ed.), John Wiley & Sons, 1976
- "*Electrical Circuit Analysis Using PSpice*", under review with Macmillan Press, 2000

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