Computer Analysis of Radiative Heat Transfer In A Rectangular Furnace

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Abstract

An idealized mathematical model of a furnace is a rectangular box. The box forms a complete enclosure. It consists of three pairs of oppositely identical plane surfaces. Under the effect of radiative heat transfer, these surfaces interact with each other either as a pair of parallel surfaces, or as a pair of mutually perpendicular surfaces.

In the computation of radiative heat transfer, an important procedure is the evaluation of shape factors. Due to the mathematical complexity of many shape factors, traditionally these are presented in the form of figures, which consist of non-dimensional horizontal and vertical axes, and a family of curves with non-dimensional geometric parameter. When a specific geometric configuration is given, the corresponding shape factors can be looked up from the figures. However, such a graphical method is inherently subject to inaccuracy and human error.

Another mathematical complication arises from the nature of a complete enclosure. Due to the fact that interactive radiative heat transfer occurs between any two surfaces within the enclosure, the analysis requires the solution of a set of simultaneous algebraic equations. In the most general case, this means the solution of six equations with six unknowns.

The present study provides a computer analysis of the stated problem. The computer program can be used to evaluate the shape factors, perform the related computation, and then solve the resulting algebraic equations. In the program, the geometric dimensions and physical properties can be changed easily. Also, the final results are more accurate than the traditional hand calculation.

Introduction

On the problem of heat transfer by radiation between two parallel rectangular surfaces, or between two perpendicular surfaces with a common side, the traditional method is to determine graphically the shape factor as a function of geometric dimensions [1], [2], [3], [4]. These diagrams are usually at most one-half page in size. The accuracy of reading is limited to about two numerical digits. Each time the geometric dimensions are changed, new values of shape factor have to be obtained from the diagrams. Additional computations involving surface areas, temperatures, emissivities, radiation energy, solution of simultaneous algebraic equations, etc., have to be performed manually. The process is time-consuming and is subject to the accumulation of numerical inaccuracy.

In the present study, a computer program in FORTRAN has been developed. The program can be executed using a desk-top computer based FORTRAN compiler. With the educational purpose in mind, a detailed analysis of the performance of a furnace can be obtained. The physical factors to be considered are: the length, width, and height of the furnace, the temperature and emissivity of the surfaces, and the amount of heat exchange among the various surfaces.

Mathematical Analysis

Notation

A_i Area of surface i, m^2
B_{ij} Element in the coefficient matrix [B]
C_i Element in the constant matrix [C]
$E_{bi} = \sigma T^4$ Black body emissive power of surface i, W/m^2
$E_i = \varepsilon_i E_{bi}$ Emissive power of gray surface i, W/m^2

$F_{i\text{-}j}$ Shape factor between surfaces i and j
G_i Irradiation on a surface, W/m^2 (thermal radiation incident on a surface)
J_i Radiosity of surface i, W/m^2 (all the radiation leaving a surface. Includes the original emission and the reflected energy)
q_i Net radiant energy leaving surface i, W
s_1 = depth/height
s_2 = width/height
s_3 = width/depth
s_4 = height/depth
α_i Absorptivity of surface i
δ_{ij} Kronecker delta, =1 when i=j, =0 when $i \neq j$
ε_i Emissivity of surface i
ρ_i Reflectivity of surface i
σ Stefan-Boltzman constant, $=5.6697\times10^{-8}\ W/m^2\text{-}K^4$
τ_i Transimissivity of surface i, = 0 for most solid surfaces except glass

1. The Shape Factor Between Two Finite, Parallel, Opposite Rectangles: $F_{1\text{-}2}$

With the geometric arrangement shown in Figure 1, the shape factor $F_{1\text{-}2}$ is given by the following equation [2]:

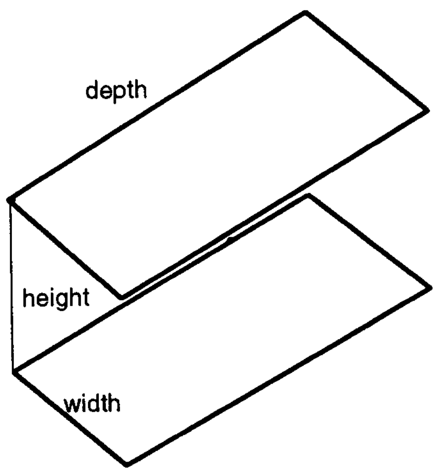

Figure 1. Parallel Surfaces

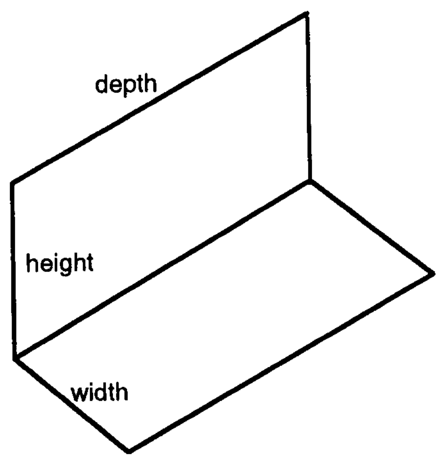

Figure 2. Perpendicular Surfaces

$$F_{1\text{-}2} = \frac{1}{\pi}\left\{\frac{1}{s_1 s_2}\ln\left[\frac{(1+s_1^2)(1+s_2^2)}{(1+s_1^2+s_2^2)}\right] - \frac{2}{s_1}\tan^{-1}s_2 - \frac{2}{s_2}\tan^{-1}s_1 + 2\sqrt{1+\frac{1}{s_1^2}}\ \tan^{-1}\left(\frac{s_2}{\sqrt{1+s_1^2}}\right) + 2\sqrt{1+\frac{1}{s_2^2}}\ \tan^{-1}\left(\frac{s_1}{\sqrt{1+s_2^2}}\right)\right\} \tag{1}$$

2. The Shape Factor Between Two Perpendicular Rectangles With A Common Edge: $F_{3\text{-}4}$

With the geometric arrangement shown in Figure 2, the shape factor $F_{3\text{-}4}$ is given by the following equation [2]:

$$F_{3\text{-}4} = \frac{1}{\pi s_3}\left\langle s_3\tan^{-1}\left(\frac{1}{s_3}\right) + s_4\tan^{-1}\left(\frac{1}{s_4}\right) - \sqrt{s_3^2+s_4^2}\ \tan^{-1}\left(\frac{1}{\sqrt{s_3^2+s_4^2}}\right) + \frac{1}{4}\ln\left\{\left[\frac{(1+s_3^2)(1+s_4^2)}{(1+s_3^2+s_4^2)}\right]\times\left[\frac{s_4^2(1+s_3^2+s_4^2)}{(1+s_4^2)(s_3^2+s_4^2)}\right]^{s_4^2}\times\left[\frac{s_3^2(1+s_3^2+s_4^2)}{(1+s_3^2)(s_3^2+s_4^2)}\right]^{s_3^2}\right\}\right\rangle \tag{2}$$

3. The Amount Of Radiative Heat Flow Away From A Surface: q_i

For a single surface at temperature T_i, if the surface is an idealized black surface, which is a perfect absorber as well as a perfect emitter of radiative energy, the total radiative energy leaving the surface is given by $q_i = E_{bi} = \sigma T_i^4$. If the surface is a gray surface, as usually is the case, the total radiative energy leaving the surface is given by $q_i = E_i = \varepsilon_i \sigma T_i^4$. Notice that the radiative energy transfer is proportional to the fourth power of temperature, hence the relation is non-linear. In order to simplify the mathematical procedure, it is easier to solve for the radiative energy transfer first, then the surface temperature can be determined when necessary.

For the practical problem of radiative heat transfer in a rectangular furnace, several surfaces exchange radiative energy with one another.If the amount of heat radiates away from a surface is assigned a positive value, then the net flow of heat from a surface is given by the following relations [3]:

$$q_i = A_i (J_i - G_i) = \frac{A_i \varepsilon_i}{1-\varepsilon_i} (E_{bi} - J_i) = \sum_{j=1}^{6} A_i F_{i-j} (J_i - J_j) \tag{3}$$

Where

$$E_{bi} = \sum_{j=1}^{n} \frac{\delta_{ij} - (1 - \varepsilon_i) F_{i-j}}{\varepsilon_i} J_i \tag{4}$$

4. Radiative Heat Transfer In A Rectangular Furnace

A furnace, as shown in Figure 3, can be considered as a rectangular box consists of gray, black or adiabatic surfaces. Equation (4) can be set up for each surface, hence there are six equations with six unknowns, these are: $J_1, J_2, ..., J_6$. These equations can be represented by the matrix equation given below:

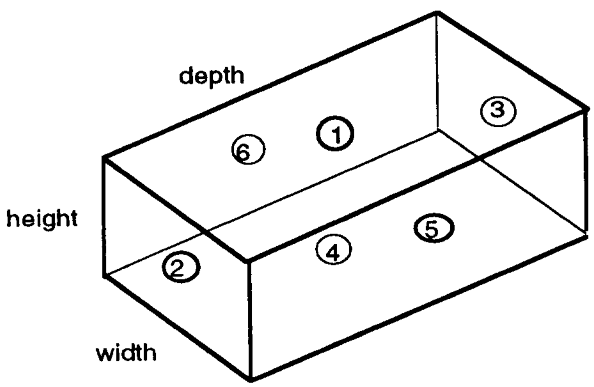

Figure 3. The Rectangular Furnace

$$[B]\,[J] = [C] \tag{5}$$

where [B] and [C] are the constant matrices. For an adiabatic surface, the element in the coefficient matrix [B] is given by the following:

$$B_{ij} = \delta_{ij} - F_{i-j} \tag{6}$$

For a black or a gray surface, the element in the matrix [B] is given by the following expression:

$$B_{ij} = \frac{\delta_{ij} - (1 - \varepsilon_i) F_{i-j}}{\varepsilon_i} \tag{7}$$

The elements in the constant matrix [C] depend on whether the surfaces are adiabatic, gray or black. For an adiabatic surface, C_i=0. For a gray or a black surface, $C_i = E_{bi}$ where E_{bi} is given by Eq.(4).

On the basis of the previous analysis, one algebraic equation can be set up for each surface, with the six radiosities J_i as the unknowns. For example, if the top surface is assumed to be gray or black, the corresponding equation is given as follows:

$$\frac{1-(1-\varepsilon_1)F_{1-1}}{\varepsilon_1}J_1 + \frac{0-(1-\varepsilon_1)F_{1-2}}{\varepsilon_1}J_2 + \frac{0-(1-\varepsilon_1)F_{1-3}}{\varepsilon_1}J_3 + \frac{0-(1-\varepsilon_1)F_{1-4}}{\varepsilon_1}J_4 + \frac{0-(1-\varepsilon_1)F_{1-5}}{\varepsilon_1}J_5 + \frac{0-(1-\varepsilon_1)F_{1-6}}{\varepsilon_1}J_6 = \sigma T_1^4 \qquad (8)$$

If another surface, such as the front, is assumed to be adiabatic, the corresponding equation is given as follows:

$$-F_{2-1}J_1 + (1 - F_{2-2})J_2 - F_{2-3}J_3 - F_{2-4}J_4 - F_{2-5}J_5 - F_{2-6}J_6 = 0 \qquad (9)$$

In order to solve these six equations for the six unknowns, a computer subroutine which is coded on the basis of Gaussian elimination can be used.

Finally, with the radiosities determined, the temperature of an adiabatic surface within the enclosure can be computed using the following relation:

$$T_i = \left(\frac{J_i}{\sigma}\right)^{1/4} \qquad (10)$$

For the gray or black surfaces, the amount of heat transfer can be computed using the equation given below:

$$q_i = A_i \sum_{j=1}^{6} (\delta_{ij} - F_{i-j}) J_j \qquad (11)$$

5. The Computer Program

The FORTRAN program consists of a main program and the following four subprograms: (a) evaluation of the shape factor F_{1-2}, (b) evaluation of the shape factor F_{3-4}, (c) computation of elements for matrices [B] and [C]. (d) the Gaussian elimination. The flowchart is shown in Figure 4.

In order to compute the various shape factors, repeated reference to the subprograms F_{1-2} and F_{3-4} are necessary. The subprogram MATRIX computes the various elements of matrix [B] and matrix [C]. The main program contains statements on the input of data and the output of computed results. Repeated execution of the entire program package can be done when a set of new data is specified.

Numerical Results

Example 1: Input data:
height=2.50 m, depth=10.00 m, width=5.00 m, T_1=400 °C, T_2=300 °C, T_3=200 °C, T_4=400 °C, T_5=600 °C, T_6=800 °C, ε_1=0.300, ε_2=0.400, ε_3=0.500, ε_4=1.000, ε_5=0.600, ε_6=0.400

Computed results: Shape factor F_{i-j} :

0.00000	0.07865	0.07865	0.50899	0.16686	0.16686
0.31460	0.00000	0.03618	0.31460	0.16731	0.16731
0.31460	0.03618	0.00000	0.31460	0.16731	0.16731
0.50899	0.07865	0.07865	0.00000	0.16686	0.16686
0.33371	0.08365	0.08365	0.33371	0.00000	0.16527
0.33371	0.08365	0.08365	0.33371	0.16527	0.00000

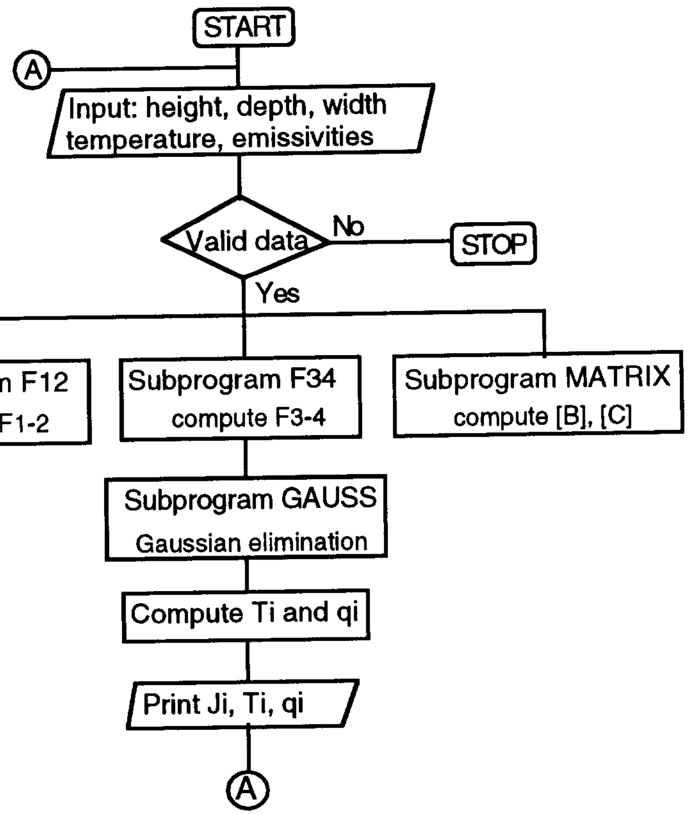

Figure 4. Program Flowchart

Matrix [B]:

3.33333	-0.18352	-0.18352	-1.18764	-0.38933	-0.38933
-0.47190	2.50000	-0.05427	-0.47190	-0.25096	-0.25096
-0.31460	-0.03618	2.00000	-0.31460	-0.16731	-0.16731
0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
-0.22247	-0.05577	-0.05577	-0.22247	1.66667	-0.11018
-0.50057	-0.12548	-0.12548	-0.50057	-0.24790	2.50000

Matrix [C]: in W/m^2

11641. 6118. 2842. 11641. 32955. 75197.

Matrix [J] in W/m^2:

16922. 14813. 11781. 11642. 27108. 39821.

Heat flow q_i in W:

q_1= -113144 q_2= -72457 q_3= -111738 q_4= -511536 q_5= 219258 q_6= 589607

Example 2: Input data:

height=5.00 m, depth=20.00 m, width=10.00 m, T_4=200 °C, T_6=400 °C, surfaces 1, 2, 3, and 5 are adiabatic.
ε_4=1.000 (black), ε_6=0.400 (gray)

Computed results: Shape factor $F_{i\text{-}j}$:

0.00000	0.07865	0.07865	0.50899	0.16686	0.16686
0.31460	0.00000	0.03618	0.31460	0.16731	0.16731
0.31460	0.03618	0.00000	0.31460	0.16731	0.16731
0.50899	0.07865	0.07865	0.00000	0.16686	0.16686
0.33371	0.08365	0.08365	0.33371	0.00000	0.16527
0.33371	0.08365	0.08365	0.33371	0.16527	0.00000

Matrix [B]:

1.00000	-0.07865	-0.07865	-0.50899	-0.16686	-0.16686
-0.31460	1.00000	-0.03618	-0.31460	-0.16731	-0.16731
-0.31460	-0.03618	1.00000	-0.31460	-0.16731	-0.16731
0.00000	0.00000	0.00000	1.00000	0.00000	0.00000
-0.33371	-0.08365	-0.08365	-0.33371	1.00000	-0.16527
-0.50057	-0.12548	-0.12548	-0.50057	-0.24790	2.50000

Matrix [C] in W/m^2:

0. 0. 0. 2842. 0. 11642.

Matrix [J]: in W/m^2:

3906. 4092. 4092. 2842. 4064. 6821.

Heat flow q_i in W:

q_1=0. q_2=0. q_3=0. q_4=-321345. q_5=0. q_6=321345.

Temperature T_i in °C:

T_1=239.18 °C, T_2=245.17 °C, T_3=245.17 °C, T_5=244.27 °C

Summary

A FORTRAN computer program has been developed for the study of radiation heat flow in a rectangular furnace. A desk top computer with the necessary compiler is to be used for the numerical computation. The program greatly facilitates the speed and accuracy of computation. Various factors affecting the operation of a furnace can be investigated. These factors include the physical dimensions, the temperature of the surfaces, and the emissivity of the materials.

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