# Linkage Analysis and Simulation Using MATLAB and Working Model

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#### Abstract

MATLAB files are developed to implement kinematics equations in a textbook to analyze three different kinds of mechanisms, and to animate their motion. Students can modify these MATLAB files for mechanisms of different link dimensions and kinematic variables, and therefore can perform functions similar to those of programs developed by the textbook's author. With MATLAB source codes, however, students can better understand the approach presented in the textbook. Working Model files are also developed to simulate these linkages for data validation and for better visualization.

#### **Background**

The textbook, *Design of Machinery* by Robert Norton [Norton 1998], used in MEEN 440 Mechanism Design and Analysis at Department of Mechanical Engineering, North Carolina A&T State University, has the leading market share in the field. The book has an attached CD containing programs for fourbar and slider-crank analysis. However, as source codes are not available, students can not effectively relate the analytic approach presented in the textbook to these programs. Homework problems are expected to be solved with calculators according to the procedures presented in the book. The long and tedious process is error prone and often frustrates students. Furthermore, programs of other mechanisms like inverted slider-crank are not available in the CD to verify results of homework problems.

#### Introduction

MATLAB [MATLAB User's Guide, 1998] is a technical computing environment for high performance numerical computation and visualization. MATLAB integrates numerical analysis, matrix computation, and graphics in an easy-to-use environment. It is an interactive system which enables the user to solve many numerical problems in a fraction of the time that it takes to write a program in a language such as FORTRAN or C. In some regards, MATLAB is a higher language than those common programming languages, meaning that the user spends less time on formalism and syntax, and therefore more time to think.

MATLAB's graphics system provides a variety of sophisticated techniques for presenting and visualization information, and therefore provides a good environment for kinematic analysis and simulation. Biswas and Kinzel [1997] have developed kinematic analysis programs based on MATLAB.

Graphics capability of MATLAB programs and Norton's programs, as illustrated in Figure 1, are limited by using sticks (line segments) to model links, and this representation is often too abstract to some students. Working Model [Working Model User's Manual, 1996], a Windows based motion analysis software, can be used as an effective tool for simulation and analysis of realistic mechanical systems. A user can sketch a mechanical system using a variety of simple geometric primitives,

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then sketch additional constraints (joints, springs, shock absorbers, pulleys, cables, and dampers) and actuators (cylinders and motors). With a click of the mouse, Working Model uses its simulation engine to set the model in motion.



Figure 1 The Norton's Four-Bar Program



Figure 2 A Working Model File of a Four-Bar

Working Model applies Newton's law along with physical constraints and external forces to calculate the internal forces and accelerations of each rigid body. Using numerical integration, the velocity and position of each body can be calculated. Working Model gives numerical results, such as force, velocity, acceleration, position, energy, etc., in digital, bar or graph form, as illustrated in Figure 2. Working Model can also generate an output data file of kinematic variables of each simulation.

# Analysis and Simulation Files

Three MATLAB files, one shown in Figure 3, are written to perform position, velocity, and acceleration analysis of three types of mechanism: fourbar, slider-crank, and inverted slider-crank. These

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files are developed based on formulation presented in Chapters 4, 6, and 7 of Norton's book. Each MATLAB file can simulate the linkage motion graphically in a complete cycle, and can calculate position, velocity, and acceleration at each frame of animation. The analysis results of these files have been verified with Norton's program. Corresponding Working Model files, one shown in Figure



Figure 3 A MATLAB File of A Four-Bar Linkage

4, are also developed for validation and for better visualization.

Linkage simulation stops at singular positions. In MATLAB files, this happens when the kinematic variable, a joint angle for example, can not be solved, and in Working Model files, it is because the torque required is too large.

For a fourbar linkage, the singularity happens when a non-Grashof linkage reaches its toggle position, or a limit position, and the fourbar linkage forms a triangle, as shown in Figure 5. In the Working Model file, the simulation will be halted before the limit position, as the torque required is increasing very rapidly causing the numerical integrator of the Working Model to stop.

If link's weight is negligible, the coupler link is a two-force member. When the linkage is at the limit position, the force transmitted to the output link will pass through its fixed pivot. Therefore, there is no torque to rotate the output link because the moment arm is zero.

For an inverted slider-crank, the singularity happens when the effective length of coupler link, the



distance between the moving pivot and the center of the slider, is zero, as shown in Figure 6 and 7. In this singularity position, the loop closure equation in Norton's book has only one unknown variable left with two equations, and therefore can not be solved.

This singularity can also be explained by mechanics which is used by Working Model. The force at the moving pivot joint can be decomposed to a component parallel to the coupler link, which is responsible for sliding motion, and a component normal to the coupler link, which is responsible for rotation of the output link. When the effective link length is zero, there is no torque to rotate the output link because the moment arm is zero.



Figure 5 A Four-Bar in a Singular Position



Figure 6 An Inverted Slider-Crank at a Singular Position

# **Discussion**

Working Model simulation is based on kinetics with inertia involved. On the other hand, MATLAB simulation files developed are based on kinematic equations in Norton's book, with no concern of force and torque. Therefore, comparison of position and velocity of constant velocity motion can be easily done, but comparison of constant acceleration is not possible except at the beginning position of a simulation motion.

Feedback from students has been positive. With MATLAB files, students can finish their homework assignments faster with better understanding. Their feedback helped guide the direction for further development - a Microsoft Word based electronic book with hyper-linked MATLAB files and Working Model files. The integrated electronic book should ease the learning of this important subject.

# **Acknowledgment**

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