

# Understanding Dynamic of Machinery with Friction

## Through Computer Simulation

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### **Abstract**

One of the main areas in the Mechanical Engineering Education is Dynamics. This paper presents how Computer Simulation has been used in the course EML 6930, Dynamics of Machinery with Friction, to improve student's understanding of the friction phenomena. One of the Design and Simulation projects developed during the course is presented. The project asked the students to design and model an apparatus that demonstrate Slip-Stick. A Karnopp [1] Friction Model with a mathematical simplification was introduced. The simulation was performed using Matlab and Simulink.

### **Introduction**

The friction forces are one of the most dominant phenomena that concern Design Engineers. You can think about it as "They are always going to be there". If any Mechanical Engineering program intends to prepare students to understand, design, and control their surrounding reality, it should be able to give them a basic understanding about Friction.

The course EML 6930, Dynamic of Machinery with Friction, in the University of South Florida, was created with that intention.

The coursework includes two design projects and two computer modeling and simulation projects. The first pair of Design-Simulation projects asked the student to design, model, and simulate an apparatus that demonstrate the Slip-Stick phenomenon. The second pair of projects asked to design, model, and simulate a vibratory conveyor, based on the Hunt-Crossley contact damping [2], and some empirical recommendations based on research and manufacturing experience [3,4].

One of the objectives for this project was to estimate system parameters based on dynamic behavior. Then, to understand how those parameters affect the Stick Slip. Through computer simulation, the students were able to "see" how each parameter works. They also could appreciate how position, velocity, acceleration, and friction force change during the experiment. This task would be almost impossible to accomplish by direct measurement (in a fair small amount of time or with all the flexibility).

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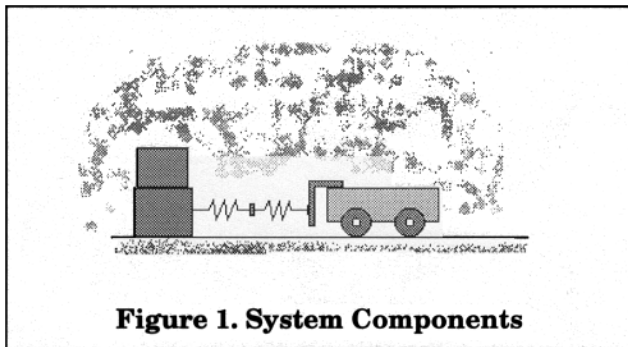
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# Project Development

The following lines describe the procedure used to model, simulate, and analyze the behavior of the apparatus.

## 1. System Description

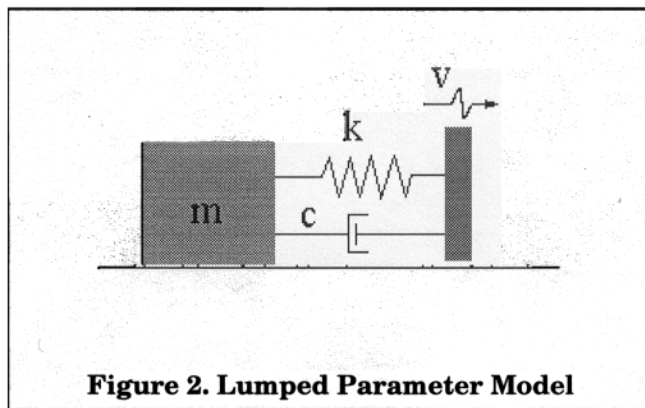
The system components are a block, two real springs in series, a drive with linear constant speed, and a rough surface. The block is connected to the drive through the spring. The drive pulls it, inducing motion between the fixed rough surface and the block. Figure 1 illustrates the system components.



## 2. Lumped Parameter Model

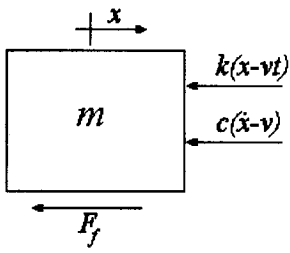
The real system is going to be modeled using a lump parameter model (Figure 2), where the following assumptions are stated:

- The driving element moves at constant speed, and it has infinite mass.
- The spring is linear, and it has the same constant both in extension and compression.
- The spring has damping as an inherent property.
- The surface is rough, and its friction coefficient can be modeled using the Coulomb Friction Model (Constant Dynamic Coefficient of Friction)



### 3. Free Body Diagram and Equation of Motion

Now, the free body diagram (Figure 3) is presented, and the equation describing its behavior:

Free Body Diagram	Equation of Motion
 <p data-bbox="217 546 618 582"><b>Figure 3. Free Body Diagram</b></p>	$m\ddot{x} = -k(x - vt) - c(\dot{x} - v) - F_f$

### 4. Parameters and Estimation Methods

Parameter	Method of Estimation	Observations	Results
Mass	Weighting	$m = 0.47Kg$	$m = 0.47Kg$
Stiffness	Deformation	$m = 0.11Kg$ $\Delta = 0.95cm$	$K = \frac{m \times g}{\Delta} = 1.14 N/cm$
Damping Constant	Logarithmic Decrement	$m = 0.14Kg$ $\frac{x_1}{x_2} = 10$	$\zeta = \frac{\ln\left(\frac{x_1}{x_2}\right)}{\sqrt{4\pi^2 + \ln^2\left(\frac{x_1}{x_2}\right)}} = 0.36$ $W_n = \sqrt{\frac{K}{m}} = 28.5 \text{ rad/sec}$ $c = 2\zeta W_n m = 2.87 \times 10^{-2} N/cm / \text{sec}$
Coefficients of Friction	Surface Inclination	$\theta_s = 27.5^\circ$ $\theta_D = 16.5^\circ$	$\mu_s = \text{Tan}(\theta_s) = 0.52$ $\mu_D = \text{Tan}(\theta_D) = 0.30$
Driving Speed	Kinematics Fundamentals	$\Delta X = 12.1cm$ $\Delta T = 10 \text{ sec}$	$V = \frac{\Delta X}{\Delta T} = 1.21 \text{ cm/sec}$

## 5. SIMULATION MODEL

The simulation model has been created using Matlab® and Simulink®[5]. The friction model is based in Karnopp [1], but a mathematical simplification has been introduced. It basically express the friction forces as an analytical function of the Sum of External forces, Normal force, and Sliding Velocity. The parameters are the Static and Dynamic Coefficient of Friction, and the Minimum Velocity. That analytical representation is expressed as follows:

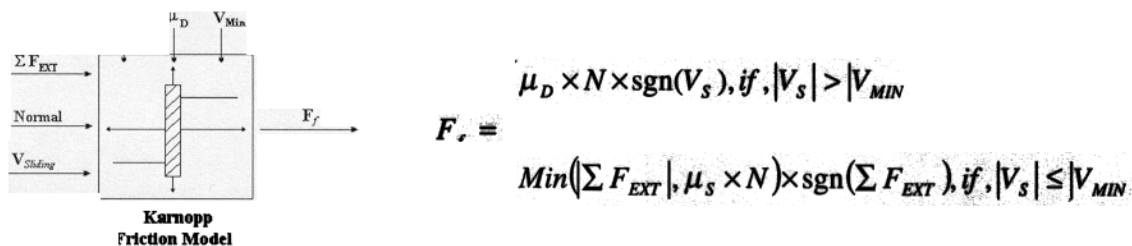


Figure 4. Simulink KFM Block

A new Simulink library component has been created to implement the Karnopp Friction Model, and it can be seen in Figure 4. A logic-gate type of design was used to decide what output is selected. The block design is presented in Figure 5. The system simulation block diagram is shown in Figure 6.

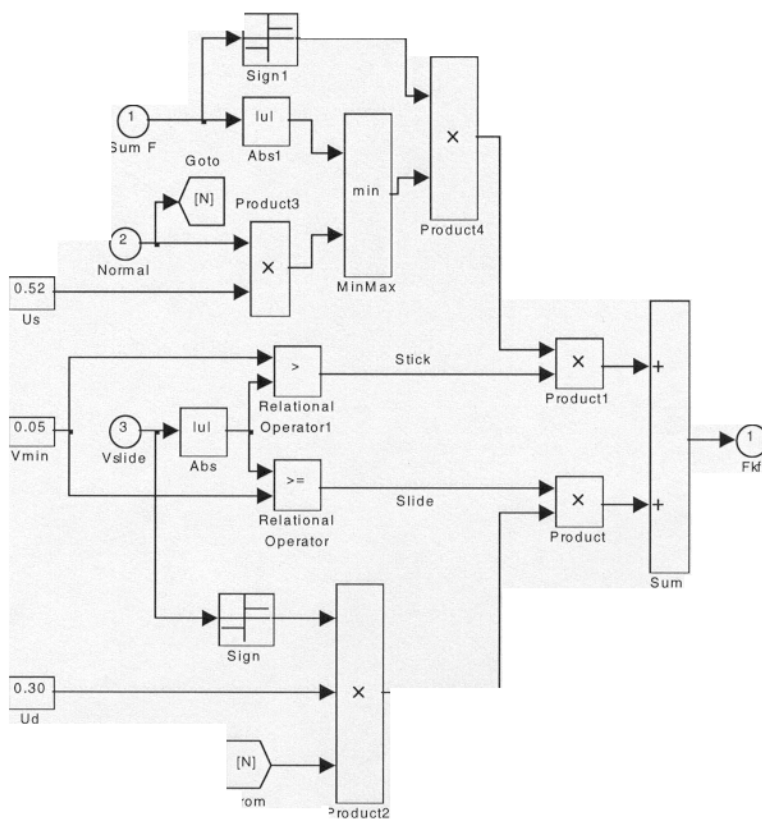
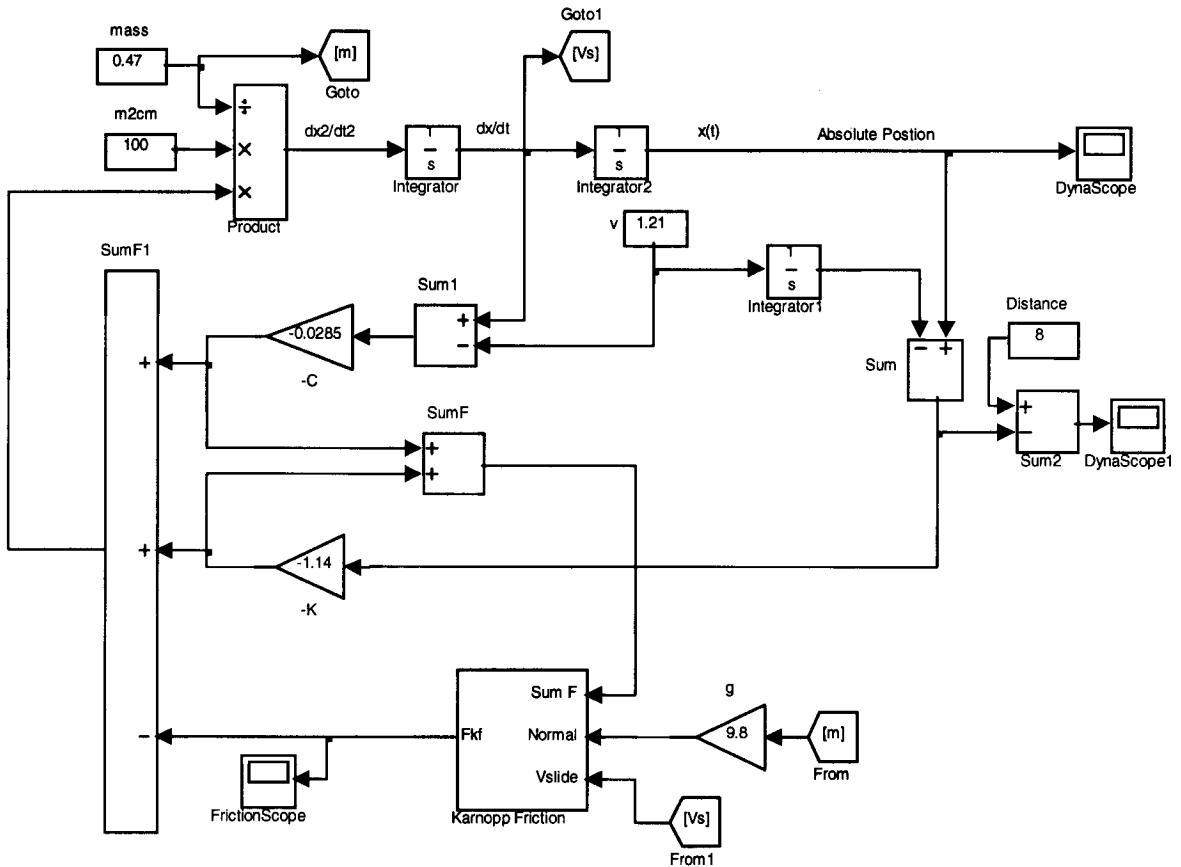


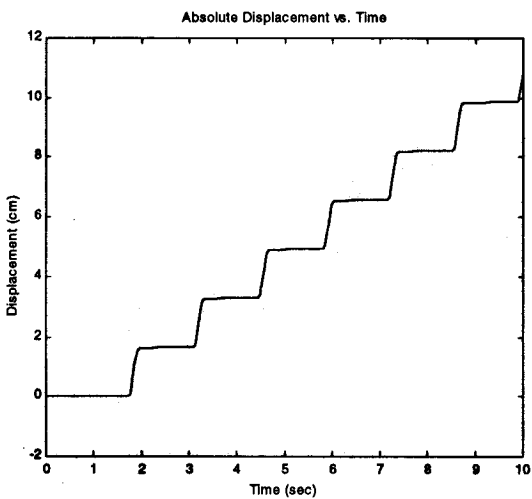
Figure 5. Karnopp Friction Model Block Diagram



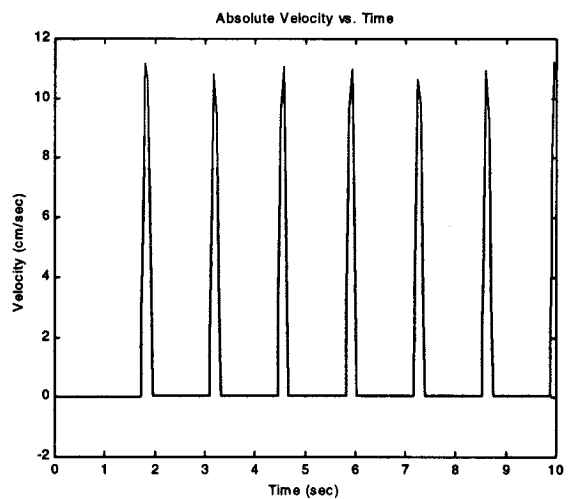
**Figure 6. System Block Diagram**

## 6. Simulation Results

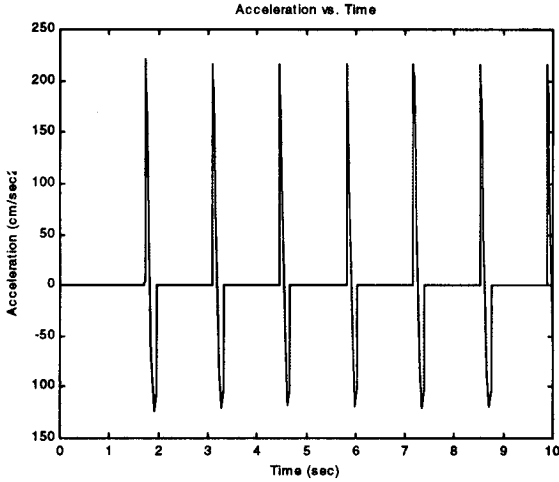
The following simulation results were obtained:



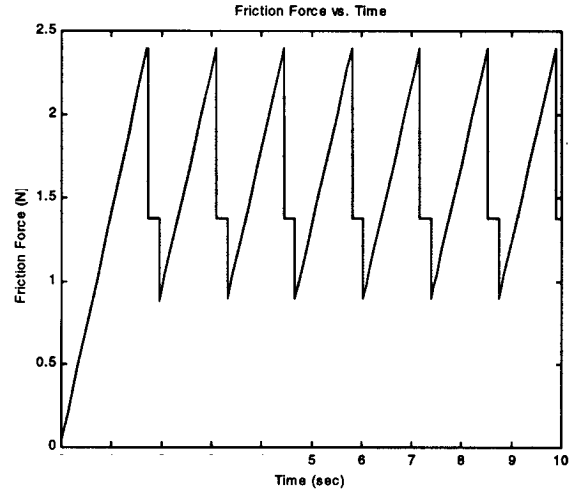
**Figure 7. Absolute Displacement versus Time**



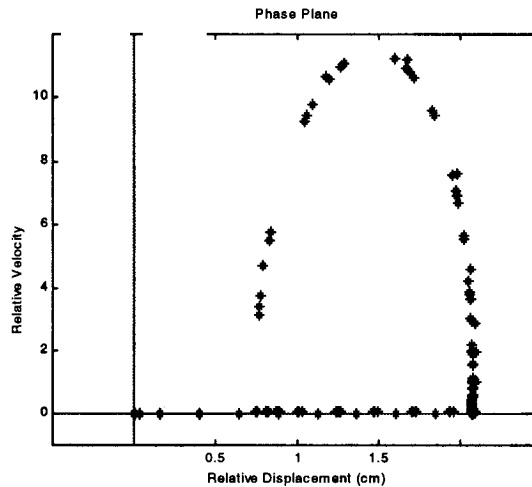
**Figure 8. Absolute Velocity versus Time**



**Figure 9. Acceleration versus Time**



**Figure 10. Friction Force versus Time**



**Figure 11. Phase Plane (Relative Motion)**

## 7. Comparison with Experimental Data

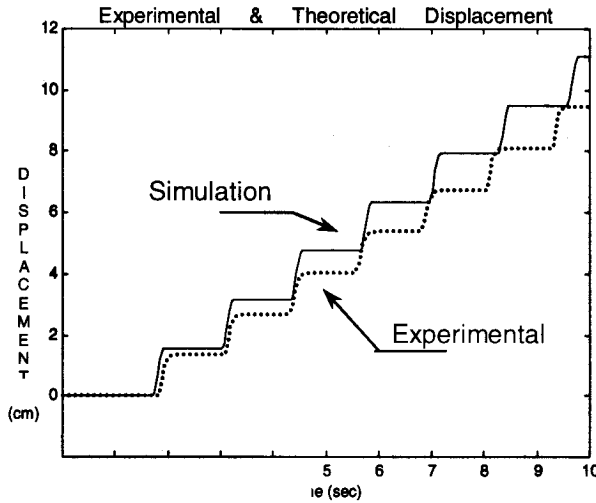
During experimentation, two types of measurement were taken: Slip-Stick Amplitude and Frequency. The amplitude was 1.35cm, and the frequency was 0.82 Hz. The simulation model gave us 1.60 cm amplitude and 0.75 Hz frequency. Figure 12 shows a superposition of both responses.

Three facts can be considered as the main responsible for the difference between the model and the experiment:

The friction model did not include change on the Dynamic coefficient of Friction as a function of the sliding velocity.

Some parameter estimation procedures, for example, logarithmic decrement for the damping constant, are not as precise as required unless we use more sophisticated measurement devices.

We assumed a linear spring, with identical stiffness both in tension and compression. That is not necessarily true, and may also be added to the change in the damping constant in both directions of motion.



**Figure 12. Results Comparison**

It is important to notice that the budget required for this project never exceeded sixty dollars, that is very low considering the amount of concepts that the student applied and modified during the implementation.

## Conclusions

From this class experience can be concluded that:

The teaching of dynamic phenomena, for example, friction, can be improved using computer simulation tools, such as Simulink, because it allows students to modify many operating conditions for a given device, and observing the results almost immediately. It also reduces the cost of laboratory equipment, not always as flexible as required.

The type of projects that involves a prototype construction and computer simulation is suitable for Mechanical Engineering education, at low cost, and with very rewarding results from the educational point of view.

Mechanical Engineering students develop a deeper understanding of friction phenomena if they have to deal with a model building task. It will enhance not only their computer skills but also their engineering analysis capabilities.

The coulomb friction model can be implemented in a computer simulation avoiding the discontinuity at zero velocity by means of the Karnopp friction model. It also can be added a mathematical simplification that avoid the segmentation study. That means that we do not have to worry about developing equations for friction forces for slipping and sticktion in a separate way.

## References

- [1] Karnopp, Dean. (1985) "Computer Simulation of Stick-Slip Friction in Mechanical Dynamic Systems", *ASME Journal of Dynamic Systems, Measurement, and Control*, Vol. 107, pp. 100-103.
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