Observations after Taking a Pre-Calculus Course

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Abstract

The first author took Math 1710 Pre-Calculus I during the 2012-13 academic year at Tennessee Technological University. The second author was the instructor of record. This paper describes the first author's experience taking pre-calculus for the first time. The Math 1710 course is similar to a course in college algebra—trigonometry is covered in Math 1720 Pre-Calculus II. In addition to participating in the course for the semester, the first author observed the in-class aspects of the course and talked to students before and after class, playing the role of a nontraditional student. Additionally, the students were not aware that the first author was a fifteen-year veteran faculty member in engineering. In a manner substantially less spectacular than the immersion sports journalism of the late George Plimpton, the first author even managed to make A-marks on some quizzes and tests! Although mostly anecdotal in the nature, the observations of both authors provided some valuable insights into the in-class aspects of the course and the course performance expectations.

Keywords

Pre-Calculus, Math Expectations, Algebra Skills, Classroom Observations

Introduction

Have you ever wondered just exactly what college students learn in a college pre-calculus course? I have, but I knew the answer would be difficult to find. Instead of this question, I decided to study a related one: have you ever wondered just exactly what goes on in a college pre-calculus course? To explore this question, I decided to informally audit a pre-calculus course at Tennessee Technological University (TTU) during the 2012-13 academic year. I teach mechanical engineering at TTU and have seen a lot of "creative" algebra and trigonometry. In fact, the math mistakes I have seen are evenly divided between arithmetic, algebra/trigonometry and higher math (calculus and differential equations). For what's it worth, the math mistakes of graduate students are in the same proportion as undergraduates.

I found a willing accomplice in Troy. Troy is a seasoned instructor in the math department. We were already collaborating on TTU's Math Success for STEM Majors (MSSM) grant funded by NSF. In addition, we were preparing a proposal for internal funding, "First-Year Engineering Math." This proposal was an adaptation and implementation of the "Introductory Mathematics for Engineering Applications" course developed at Wright State University.

We decided on some basic ground rules. First, I would essentially take the course as a student. Troy would treat me simply as a student and we would have no interaction inside or outside of class that would give any other impression. For my part, I would attend class as regularly as possible and take quizzes and tests. I would make observations of the class only to the extent that any other student could. I would talk to students before and after class in a casual manner, but I would never meet with students outside of class. For my part, I would do no homework, and instead, rely solely on the lectures and Troy's questions and answers with students. In that way, I thought I would be able to see how much pre-calculus a practicing engineer/engineering professor retained.

I had this grand dream of having a George Plimpton-type immersion experience. In my childhood, George Plimpton the journalist and writer was also George Plimpton the professional baseball and football player, boxer and golfer. He wrote about sports through his style of participatory journalism that I found intriguing. After reading his book, *Paper Lion*, about his training camp and preseason experiences with the Detroit Lions, I figured I could pluck up the courage to be a pre-calculus student. It turns out that I am no "George Plimpton."

The Course and Its Neighborhood

Tennessee Technological University (TTU) has several college algebra and pre-calculus courses. Math 1130 College Algebra (3 cr. hrs) is separate from the pre-calculus courses at TTU. It has some terminal features for students not going on to calculus. Math 1730 Pre-Calculus is a five-credit hour course covering both algebra and trigonometry. The topics in Math 1730 are covered at a somewhat slower pace in the sequence Math 1710 and 1720 Pre-Calculus I and II (both are 3 cr. hrs). I took Math 1710 Pre-calculus I. The official course description for Math 1710 is

> "Review of algebra; relations and functions and their graphs, including polynomial and rational functions; conic sections; inequalities, arithmetic, and geometric sequences and series."

The course objectives, as stated in official syllabus, are

> "Build on (not replicate) the competencies gained through the study of two years of high school algebra and one year of high school geometry. Use mathematics to solve problems and determine if the solutions are reasonable. Use mathematics to model real world behaviors and apply mathematical concepts to the solution of real-life problems. Make meaningful connections between mathematics and other disciplines. Use technology for mathematical reasoning and problem solving. Apply mathematical and/or basic statistical reasoning to analyze data and graphs. Refine the algebraic, geometric, and reading comprehension skills necessary in the study of calculus."

We used Lial, Hornsby and Schneider's book. We covered five-six chapters in this large book (it's nearly the same size as a calculus book). The principal subjects were functions and their graphs, polynomial and rational functions, exponentials, logarithms, partial fractions, and sequences and series. The Math 1710 course focuses on algebra and is coupled with Math 1720, which focuses on trigonometry. Students who are not eligible to take the faster paced combined course, Math 1730, will likely be a full academic year behind in the math requirements for engineering. A limited number of students are allowed to take Math 1710 and 1720 concurrently. These students would be a single semester behind schedule (just as the Math 1730 students would be).

Troy's Math 1710 course policy included the usual 10-point grading scale: 90-100 for an A, etc. He assigned homework regularly each day, but did not grade it. He did discuss any homework problem that a student asked about. Grades were based on three one-hour tests totaling 45% of the course grade, 10-12 short quizzes for 30% and a comprehensive, two-hour final exam for 25%. He did not allow graphing calculators for quizzes and tests, but he did allow a simple, scientific calculator, such as the TI-30IIS or Casio fx-115es. I used my thirty-year-old HP 15C with RPN notation.

Firsts in the Class

First Day of Class: The class section had 35 students (plus one) in an appropriately sized room. The classroom has the usual chalkboards (which Troy used some), but also had a projector which Troy used with a small tablet computer. The setup is shown in Figure 1. Troy is the "T" and I am the "C" in the figure. On the first day, Troy spent nearly thirty minutes talking through the course syllabus. The noteworthy quote for the day was "Don't be grade-obsessed, be learning-obsessed." I am not sure if any one wrote the quote into their notes, but it served notice that Troy was into learning. He plunged into complex numbers in the next moment.

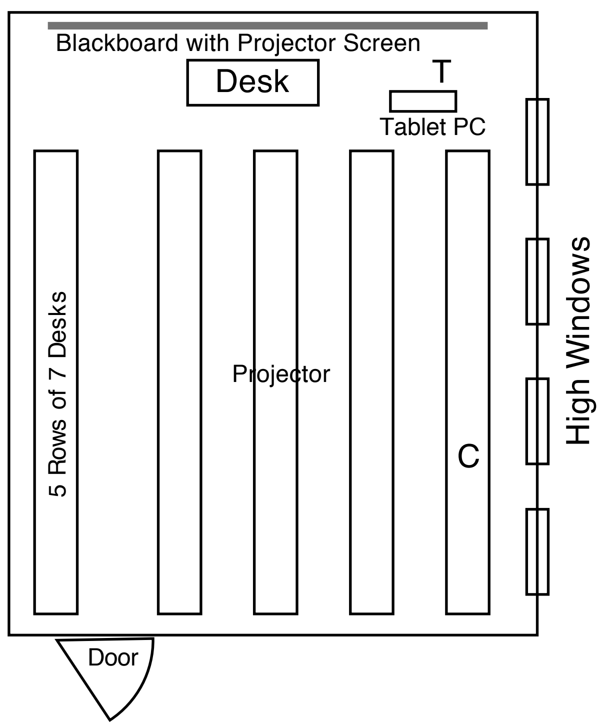

Figure 1 Room Layout

First Interactions: I came to class a few minutes early most days. During the first week of class, I talked with the students sitting around me. They were a little surprised to see an older student taking Pre-Calculus I. I told that I was going back to school and needed to start from the beginning. Quickly, I became "the old man" to four students sitting nearby. They were amazed that my calculator didn't have an equals button. I told them it was common in my youth. We would talk about how much homework they had in all their classes and how they couldn't seem to get it all done. One or two students seemed disappointed that they had to take the course because they had learned it all in high school. Others didn't mind, but only the students sitting in the front seem to be excited. I told them that I was just glad to be there.

First Quiz: The first quiz had five problems to work in ten minutes. It was given at the end of class. There was a healthy amount of chatter in the few minutes before class started. The first problem was to state the distance formula. The second was the midpoint formula. The third was to find the equation of a circle given its center and radius. All three of these problems were things that I figured everyone should have in memory. The last two problems involved simplifying complex numbers. Again, something an engineer should know. Even so, I was nervous and it took me the full ten minutes to complete the quiz and double-check my answers. One-third of the class handed in their papers at the final call. The average grade on the first quiz was 50%.

First Test: The first had ten questions. It included questions that I figured everyone should have in memory (again). Examples were sketching graphs of $y = x^3$ and $y = \sqrt{x}$, creating parallel or perpendicular lines with point location and slope information, and substituting a function into the difference quotient. As with the first quiz, it took me the full hour to complete the test. And, yes, I was nervous. The average grade was 46/100. Ouch!

First Pep Talk: I don't remember much of a pep talk after the first test. However, Troy went over the entire test in much less than an hour. He reminded the students that it was only the first test and that they had two more tests and a final exam still to do. He did encourage students to focus on ramping up their skills and their demonstration of skills and not worry so much about grades. He mentioned doing homework three times on the day he gave back the test. From talking with students before the next class, some of them seem to buy into Troy's philosophy. Others were mad either at Troy or themselves. A good student sitting next to me confided that he could do the work, but just not in the time allotted.

Quizzes and Tests

During the semester, Troy gave nine quizzes (typically on Wednesdays). As mentioned previously, he also gave three one-hour tests and a two-hour comprehensive final exam. The scores are given in Table 1. The test columns are populated in the table so that you can see that Test 1 occurred after Quiz 2, Test 2 occurred after Quiz 6, etc. The column "QT Aver" shows the average of quiz grade averages preceding a test. This simple analysis shows that the average of quiz grade averages were a good predictor of the test average for the first two tests. After Test 2, the trend breaks down. I thought the quizzes were a little easier after Test 2. Attendance for each quiz and test is also given in Table 1.

The first quiz and the first test are given in the appendix of the paper. I encourage STEM instructors to sit down and complete the quiz in 10 minutes and the test in 55 minutes to get a sense of what we experienced.

Table 1 Quiz and Test Scores (Attendance in Parentheses)

Quiz	Q Score	QT Aver	Test	T Score
1	5.0 (35)			
2	3.3 (33)			
		4.2	1	45.6 (35)
3	7.8 (33)			
4	6.0 (31)			
5	7.6 (31)			
6	6.8 (33)			
		7.0	2	69.0 (31)
7	9.6 (26)			
8	6.8 (28)			
		8.2	3	63.5 (28)
9	9.0 (24)			
			Final	51.7 (29)

Observations

Classroom: The syllabus noted that attendance was required, but the attendance was never checked. There were 35 students enrolled in the class. On the first day, there was 35 students. On day two, the count was 31. On day three, the count was 27—an ominous trend for first semester students! The attendance varied from a low of 22 to a high of 35 (first day of class and Test 1). Before the last day to withdraw from a class, the quiz day attendance averaged 32.5. After that, the quiz day attendance averaged 25 although only five students withdrew from the class.

Attendance alone isn't always a good measurement of student involvement (really student investment). I did try to measure the engagement level of the students while they were in the classroom. I counted the number of students who were verbally engaged; typically, the same 4-5 students were asking most of the questions and were laughing at Troy's attempts at humor. I also tried to count the number of students who were engaged in visual-only (nonverbal) ways: watching Troy, listening to questions and answers, copiously taking notes. Typically, this visual-only group was 8-12 students. Of course, my accounting is subjective. I was also trying to take notes and laugh or cry as needed. I rarely saw a person that was totally unplugged. When I did see someone unplugged, they were usually distracting themselves and their neighbors with their cell phones.

I noticed a correlation between the perceived engagement level and where a person sat in the room. My observations of the perceived engagement areas are shown in Figure 2. Students that I saw as less engaged or not engaged during class sat in the back and near the door. I sat in the visual-only group. In talking with students before class, it was common to be asked what happened during the previous day if a person missed. I did the same thing for the two-three times that I was absent. I received fairly accurate information when I asked questions of the visual-only group.

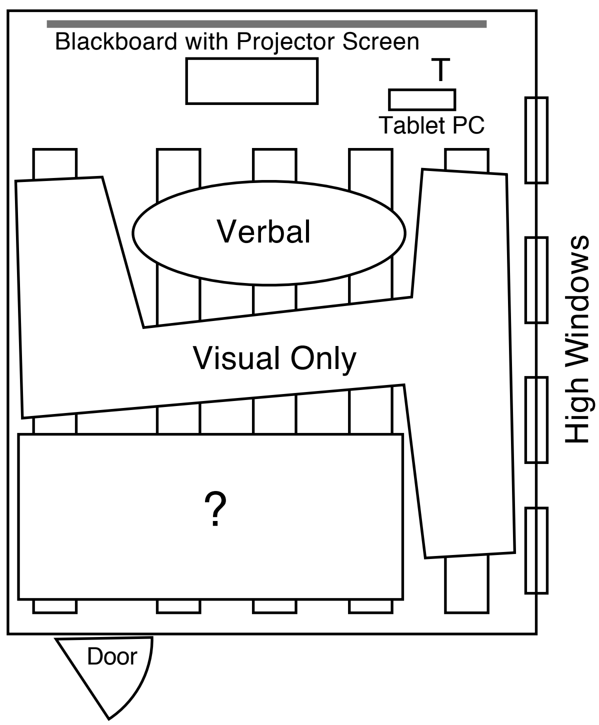

Figure 2 Perceived Engagement Areas

Personal: One of the first things I personally observed was how hard it is to take notes for 55 minutes. I found myself distracted at times. I didn't have a smart phone in those days or I would have been tempted. Because sitting in class was my only review time for the pre-calculus material, I tried to focus as best I could. I also found that I was distracted a bit by others and what they were doing. Perhaps, this distraction was due mainly to my need to observe. Another observation was that using a graphing calculator or even owning one didn't seem too much of an advantage. We couldn't use them on quizzes and tests anyway. When some of the students saw my old calculator, they poked fun of it. However, a basic calculator seemed good enough for the course.

My biggest observation was how fast time went on the quizzes and exams. I really thought I would have some margin of time at the end to check myself. After all, I have four college degrees and one of them is in applied math. Not doing homework or reviewing was my way of checking whether or not my expectations of the math an engineering should know lined up with what Troy or other college math teachers thought. The time factor was certainly part of the check. I was quite humbled by my slowness; however, like everyone else, you are good at what you repeatedly do. To a certain degree, speed is a sign that you know what you are doing. *Festina lente!*

Another observation was that I clearly don't use the course material in the level of complexity that it is taught. For my part, I couldn't find a single thing that I thought was optional material for an engineering major to see. However, there are many things that I don't use in practice. For example, when it comes to solving quadratic equations, if I don't see factors right away, I go immediately to $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The technique of completing the square is used to derive the given equation. Thus, it is certainly useful, I don't use it in practice. Like Troy, I find it disconcerting to see answers for quadratic roots such as 1.999998 and 2.499999 when factoring or completing the square might lead to 2 and 5/2. But giving such solutions might be a sign of another issue: verifying and validating results.

As another example, we graphed some really complicated rational functions: multiple zeros and inclined asymptotes, etc. In practice, such functions don't come up often in my work. Therefore, I rely on software such as MATLAB or Maple. Having written this, I plainly acknowledge that the students need to practice, practice, practice problems of varying degrees of difficulty. They need to use a variety of methods as they build their toolboxes.

The final observation was the most personal: I found that taking a freshman class—even with the limited engagement I had—helped me reconnect with life and trials of freshmen STEM majors. I have taught freshman courses in engineering graphics, computer programming and calculus. I have also taught/facilitated the "get adjusted to college life" course at our university. However, I was always on the other side of the podium! Maybe test day is not the same as a preseason NFL game with the Lions, but the 40+ hours I spent on the students' side of the podium was worth it.

In the abstract, I wrote that "I managed to make A-marks on some quizzes and tests!" This statement was a bit of underselling on my part—just as many top students seem overly worried about grades. Following the ground rule of never studying made me a little nervous. Yet, I wanted to know what pre-calculus I carry around in my head.

The Aftermath (No Pun Intended)

Before this adventure, Troy and I were already collaborating as part of an interdisciplinary team to increase the number of STEM graduates at TTU by promoting math success in pre-calculus (mostly in MATH 1730–the 5-hr pre-calculus course). The active learning strategies and other redesign efforts will certainly filter into the MATH 1710-1720 sequence. We also collaborated on an internally funded project to teach a hybrid engineering-math course to freshmen instead of

MATH 1710. This course highlighted math well beyond pre-calculus topics by including examples of how calculus and even differential equations are used in the engineering curriculum. The personal investment made in all of these project has greatly benefited our working relationship.

In the semester following the immersion experience, an engineering student, who worked as a math tutor part-time, collected math observations from a senior mechanical engineering course I taught. The student was taking the course for credit and, at the same time, kept a brief journal of the pre-calculus and calculus skills used in the senior course. Many of the MATH 1710 concepts were used in the engineering course. I also began to point out to my students the other math course connections to the senior course I was teaching: calculus, linear algebra, differential equations. I continue this practice.

Finally, I am beginning to explore the literature regarding the role of immersion experiences to enhance teaching.

Troy's Observations and Rebuttals

In my experience MATH 1710 is a difficult course to teach well. There are two main issues. The first is that this is a math that primarily services engineering students. There is frequently a disconnect between mathematicians and engineers as to what a math course should be. Mathematicians do math for the sake of math; we enjoy the challenges of the problems and developing new ways, or new tools, to solve those problems. Engineers are more concerned with learning a reliable set of tools for the most common problem types. If left to our own devices, we mathematicians would have an exceptionally rigorous course that would be very discouraging to all but the most math savvy students. Thus the challenge of teaching this course is including enough rigor for the mathematicians in the department and enough practical mathematics to produce competent engineers. The second issue that contributes to the difficulty of teaching MATH 1710 is the wide range of student abilities and interest in the course. Some sections are more homogeneous than others in the student’s math abilities and interest which make the course much more enjoyable for all. Of the two, abilities and interest in the subject, student interest level is the most important. If I have a class of interested students, that is, students that want to learn the material with the knowledge that it will be useful throughout their education and career, they will outperform a class with less interest, regardless of ability. It is a student’s interest in the subject and their education that motivates them to push themselves.

The class section that this paper deals with was populated with a wide range of math abilities. For example the MATH ACT scores of the students enrolled ranged from a low of 18 up to 27. The course is fast paced for a three credit hour course and student buy-in is difficult to achieve. As was noted earlier in the paper, students are frequently disappointed and frustrated that they are required to take this course since they usually have had advanced math courses in high school and feel that they have progressed passed this level. The first quiz and test typically have averages in the 50’s or lower. For many students, this is the clarion call to action. Students also worry unduly about being behind in their curriculum. Thus, their frustrations grow.

Student engagement is essential in a classroom, but not just for the engaged student’s benefit. Typically, every class is separated into the three groups of engagement listed above, verbal, visual only, and checked out. However, the larger the verbal group is the better the class does as a whole. The verbal group helps to increase the visual group, because students look forward to the verbal students' questions and banter. In courses with lots of questions, either based on homework or lecture, the test averages are always noticeably higher.

I primarily use a tablet computer in class for lecture notes. I project my handwriting onto the screen. In some classrooms this makes it hard for everyone to see what is going on since the classrooms were not originally designed for this media. However, I do post the lecture notes to the course website so anyone in the course can download them to compare to their in-class notes or if they miss a class meeting. Many students did not take advantage of this resource. Access typically ranged from 10 to 13 students for the tests. The first day's lecture notes had the highest access rate at 17 students accessing them. This rate potentially shows a lack of student engagement when they don't take advantage of the resources made available to them. As noted in the paper, it was difficult for Chris to keep up with the pace of class and take notes for a full 55 minutes, which would lead to an expectation of a large utilization of this resource. This is one of the reasons that I make lecture notes available to students.

Time is always a factor on math quizzes and tests. This is perhaps the biggest source of complaints about the course. It is difficult to get students to realize that they need to be fast with these skills as they will be advancing to both math and engineering courses where the skills they are learning now will be but one part in a many part problem. If it takes them five minutes to solve a simple quadratic or graph a basic function, they will not have time to complete a problem in a more advanced math course. I try to encourage them to start slow, but build speed as they build confidence in their skill set. Homework is the primary way for students to accomplish this.

In this course homework was assigned almost every class. I never took up the homework because it is not possible for me to grade the homework in a timely manner. I typically teach large course loads and in this semester I taught 18 credit hours. The first quiz and test problems were essentially the same as the homework problems. I do this to encourage students to do the homework. Typically after the first test, I will ask the class how many did all of the homework, then how many did some, and finally how many did none. Then as I go over the test, I point out which problems were directly from homework or if I had worked a problem in class. The only times I ask to see homework is when a student comes to me with questions, then I ask to show me their homework and we talk through their work.

This section of pre-calculus was a typical section. It was not the best section I have ever taught, nor was it the worst. It is illustrative of the challenges that face math instructors, engineering students, and how this course fits into the expectations of engineering faculty.

Postscript: How They Did

A total of 11 students earned grades in the ABC range, while 24 earned grades in the DFW range. At TTU, we sometimes refer to the DFW percentage as a measure of both degree of difficulty of a course and also as a measure of improvement as revise courses and try to help students move through the curriculum. The DFW percentage for the class was 68.6%. This number is typical for Math 1710 in the fall. Ten students in the class were also taking Math 1720, indicating that they were close to meeting the eligibility requirements for Math 1730. However, the results were mixed for this group: 3 of the 10 earned grades in the DFW range. Of the 24 students in the DFW range, 12 of them earned a semester grade point average of 1.5/4.0 or less.

By waiting until this year to write of the experience, we were able to see how the students progressed to graduation. A total of 17 students from the original 35 have graduated already. Only 3 of the 17 changed from a STEM major to a non-STEM major. In addition, 5 more will graduate in the coming spring term. All 5 are still STEM majors, but 4 of the 5 changed to a STEM major with reduced math requirements. The remaining 13 students are no longer enrolled at TTU; however, 4 of the 13 made grades in the ABC range.

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Appendix: Sample Quiz and Sample Test

Quiz 1 MATH 1710 Pre-Calculus I, 10 pts Name:________________________

Please write your solutions neatly and show your work in an organized manner. No credit will be given for answers without supporting work or for solutions presented in a disorganized manner which renders it difficult for me to follow. Each problem is worth the same number of points.

1. State the distance formula.

2. State the midpoint formula.

3. Find the equation of a circle with center point $(2, 3)$ and radius 3.

4. Simplify i^{71}

5. Write the quotient $\dfrac{6+2i}{1+2i}$ in standard form, $a + bi$.

END

Test 1 MATH 1710 Pre-Calculus I, 100 pts Name:______________________

Please write your solutions neatly and show your work in an organized manner. No credit will be given for answers without supporting work or for solutions presented in a disorganized manner which renders it difficult for me to follow. Each problem is worth the same number of points.

1. A diameter of a circle has endpoints $(-1,-2)$ and $(5,6)$ and a length of 10 units. Write the equation of the circle in center-radius form.
2. Give the Domain of each relation or expression and decide whether each defines a function of x.
   (a) $\{(2,4),(0,2),(2,6)\}$
   (b) $\{(5,1),(3,2),(4,9),(7,8)\}$
   (c) $x+y<3$
   (d) $xy=2$
3. Find r so that the line through $(2,6)$ and $(-4,r)$ is
   (a) parallel to $2x-3y=4$
   (b) perpendicular to $x+2y=1$
4. Write the equation of the line through the point $(-4,3)$ with slope $m=\frac{3}{4}$ in slope-intercept form.
5. Sketch the graphs of the following functions:
   (a) $y=x^3$
   (b) $y=\sqrt{x}$
   (c) $y=|x|$
6. Let $f(x)=\sqrt{4x}-3$ and let $g(x)=\sqrt{4x}+3$ find the following function operations and give the Domain for each.
   (a) $(f-g)(x)$
   (b) $(fg)(x)$
7. Find the difference quotient, $\frac{f(x+h)-f(x)}{h}$, for $f(x)=1-x^2$.
8. Let $f(x)=\sqrt{x+2}$ and let $g(x)=\frac{-1}{x}$. Find $(g\circ f)(x)$ and its Domain.
9. Suppose $f(x)$ is an odd function, that is $f(-x)=-f(x)$, and $g(x)$ is an even function, that is $g(-x)=g(x)$. Fill in the missing entries in the table.

x	-2	-1	0	1	2
$f(x)$			0	-2	
$g(x)$	0	2	1		
$(f\circ g)(x)$		1	-2		

10. A cylindrical can makes the most efficient use of materials when its height h is the same as the diameter d of its top. HINT: Volume of a cylinder is given by $V=\pi r^2 h$.
    (a) Express the volume, V, of such a can as a function of the diameter d of its top.
    (b) Express the surface area, S, of such a can as a function of the diameter d of its top.

END