Methods to Improve Students Learning in Dynamic Systems and Control Course

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Abstract

Study of Dynamic Systems involve modeling and analysis of system response to different inputs. Mathematical models of electrical and mechanical systems consist of first order and second order differential equations. To find the response of such systems, students need to solve the resulting modeling equations. Conventional methods using trial solution approach may sometimes be frustrating for the students if the model is complex. Laplace transform method do help by converting the differential equations into algebraic equations but that method is also not very helpful when it comes to solving multi degree of freedom systems. Using MATLAB simulations to solve these modeling equations make life easier for students and is a subject of this paper. While MATLAB do not replace the need for theoretical teaching, it offers a quick solution to increase students' engagement and comprehension. This method of teaching is adapted for the first time this semester and needs further investigation of its popularity and its impact on students' result but midterm evaluation feedback from students suggest that these simulations are providing an effective and enjoyable source of learning.

Keywords

Dynamic Systems, Controls, MATLAB, Simulink

Introduction

Dynamics Systems Modeling and Control is a required course in any mechanical engineering program. In some universities it is taught as two different courses: one as a Dynamical Systems course and other as Introduction to Control Systems. At Grand Valley State University this course is taught as a single course combining modeling, simulation and control of dynamic systems. It includes a theoretical part where principles of system dynamics and system control are emphasized as well as a laboratory part to emphasize theoretical knowledge with hands-on experience using real time control systems. Previously Scilab had been used in this course for simulation purpose. This semester Scilab has been replaced by MATLAB which is a much powerful software with all the documentation available to students and they may see its usage in their professional career. Another advantage of using MATLAB is the option of using Simulink which was not available in Scilab. Different ways of simulating systems by hand and in MATLAB are explored and the students are able to compare different methods and appreciate the use of software. To make the class room more active and students more engaged, different activities were performed in the class. At the end of this course, the students were able to find the dynamic response of a complex mechanical system subjected to different types of inputs relatively easily.

Course Description

Dynamics is a pre-requisite for this course. Students came with the knowledge of obtaining equations of motion of simple one degree of freedom dynamic systems. In this course, the modeling of mechanical, electrical and electro-mechanical system is covered in first few weeks. In the later weeks, the response of such systems under different external inputs is covered. For one degree of freedom system, system response can be obtained fairly easily by applying the methods used in differential equation class. For two or more degrees of freedom systems, the resulting mathematical models consists of coupled differential equation whose solution is not that straight forward. One of the methods to obtain the system response for such systems taught in this course is by using Laplace transform. Solving complex system with method of Laplace transforms is sometime frustrating for the students as it involves lot of algebra.

Method of Instruction

Traditional method for obtaining dynamic system response is based on method learned in differential equations class (solving for homogeneous and particular solution) and for complex system using Laplace transforms. This semester the author tried to teach this course in a more interesting and fun way. Simulations of dynamic systems were obtained using MATLAB/Simulink. A tutorial on MATLAB/Simulink is presented as a lab [2] in the second week of classes and the lectures are integrated with MATLAB to teach students how to obtain dynamic system response to different inputs for any system. It is easier for students to see a simple system respond to different inputs by doing their hand calculations/derivations and then compare their response with the MATLAB plots. Some in class activities with some fun competitions were also performed.

MATLAB Simulations

Example 1: Spring Mass Damper System

Given a simple one degree of freedom mechanical system:

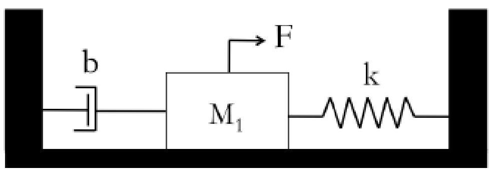

Figure 1- Spring-Mass Damper System

The equation of motion of the system is given as:

$$M\ddot{x} + b\dot{x} + kx = F(t)$$

Analytical Solution

The homogeneous solution can be obtained by solving for the roots. If the roots are real and distinct, the solution would be:

$$x_h(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

If F(t) is a unit step input , the particular solution can be found as:

$$x_p(t) = A_3$$

Solve for A_3 and then apply initial conditions on the total solution to solve for the constants A_1 and A_2. The total solution is given as:

$$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3$$

It is clear from the response that if the roots are negative (stable system), the system settles at A_3.

Numerical Solution Using Simulink

The differential equation of the system is given as:

$$M\ddot{x} + b\dot{x} + kx = F(t)$$

Rewriting the equation in a form suitable to be used in Simulink as:

$$\ddot{x} = \frac{1}{M}(-b\dot{x} - kx + F(t))$$

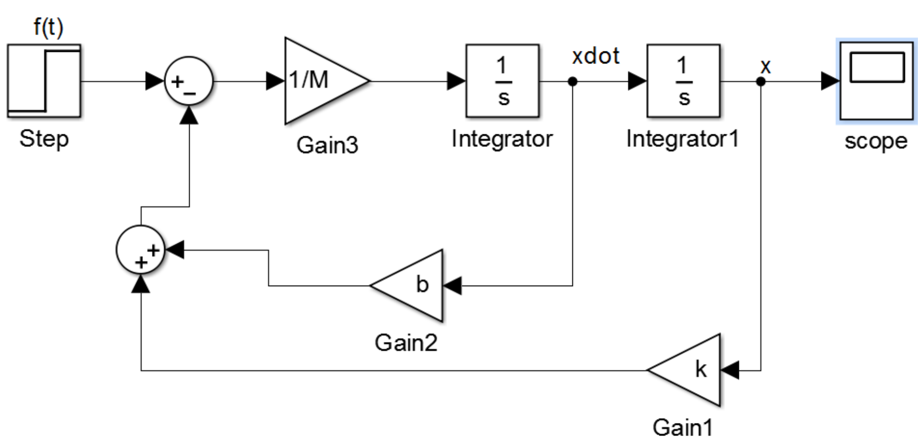

Figure 2- Simulink Model for System in Example 1

Using M=1 , b=2 and k=100, the system response for a unit step input is shown in figure 3. Even though solving this system by hand using traditional approach is not that complicated, but the students enjoys using Simulink to find the response. They can see what goes into the integrator, what comes out from the integrator and how the whole equation is making sense.

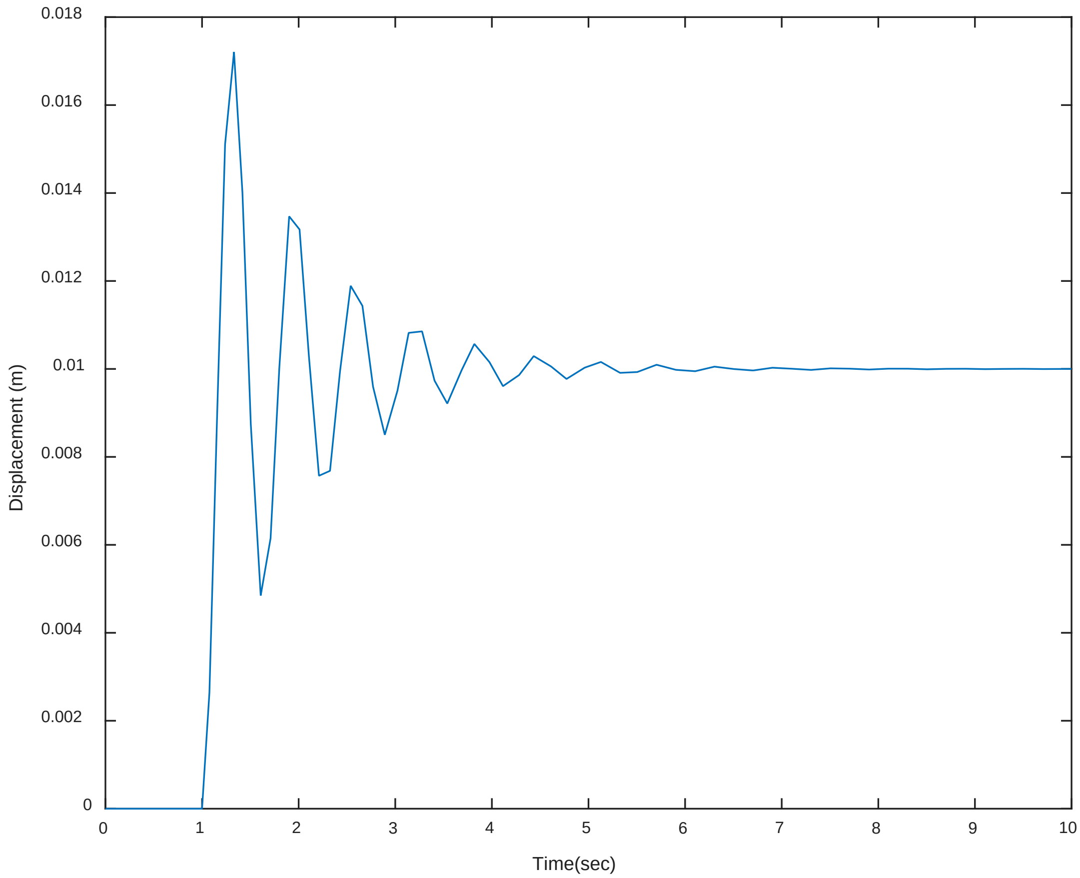

Figure 3- Step Response of System in Example 1

Example 2: Simulation of a Two Degree of Freedom Spring Mass System [4]

A two degree of system is shown in figure 4. The resulting equations of motion of the system are coupled differential equations given as:

$$m_1\ddot{x}_1 + (K_{d1} + K_{d2})\dot{x}_1 - K_{d2}\dot{x}_2 + (K_{s1} + K_{s2})x_1 - K_{s2}x_2 = 0$$
$$m_2\ddot{x}_2 + (K_{d3} + K_{d2})\dot{x}_2 - K_{d2}\dot{x}_1 + (K_{s3} + K_{s2})x_2 - K_{s2}x_1 = F$$

The complete solution by hand using Laplace transforms is shown in Appendix. As seen from appendix, finding the total solution of this system by hand is not something a student would like to do, even though it is not technically difficult but involves a lot of algebra. However, this system can be simulated in Simulink in three different ways:

a. using integrator blocks
b. using transfer function block
c. using state space block.

The simplest way is to use state space block which is presented in this paper.

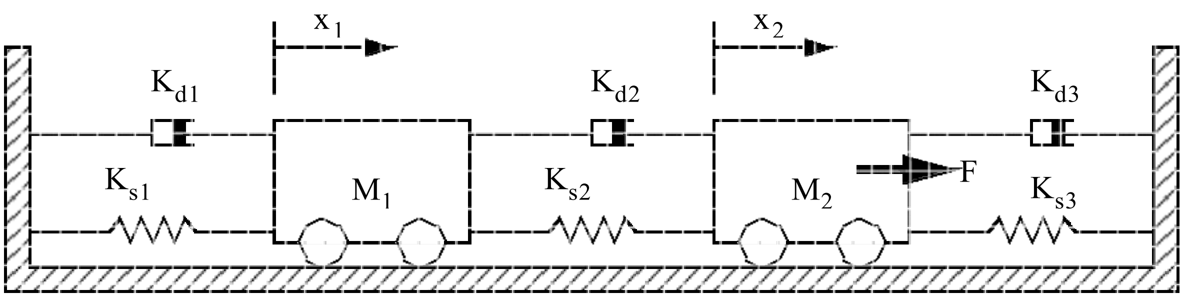

Figure 4- Two degree of freedom mechanical system-Example 2[4]

State Space Method

State Space Method converts the two second order differential equations into a set of four first order differential equations. The state space model is written in the following standard form:

$$\dot{Z} = [A]\{Z\} + [B]\{F\}$$
$$Y = [C]\{Z\}$$

where Z is a vector of states, A is a matrix of constant which contains the coefficient of states, B is a matrix containing coefficients of inputs, Y is a vector of outputs, C is the output matrix and F is a vector of inputs. Using F=0, K_{s3} =0 and all damper constants equal zero, the state space model of the system is given as:

$$\dot{Z} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-(K_{s1}+K_{s2})}{m_1} & 0 & \frac{K_{s2}}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K_{s2}}{m_2} & 0 & -\frac{K_{s2}}{m_2} & 0 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{Bmatrix}$$

$$Y = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{Bmatrix}$$

The Simulink model of the system is shown in figure 5.

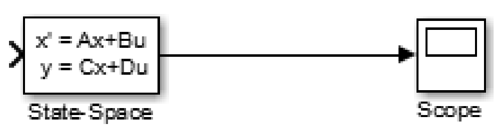

Figure 5- Simulink of the two degree of freedom system of Example 2

Using the following data [5]:

m_1=2, m_2 = 4, K_{s1}=8, K_{s2}= 4 and the initial conditions: $x_1(0) = \dot{x}_1(0) = \dot{x}_2(0) = 0, x_2(0) = 1$, the free response of the system is shown in figure 6.

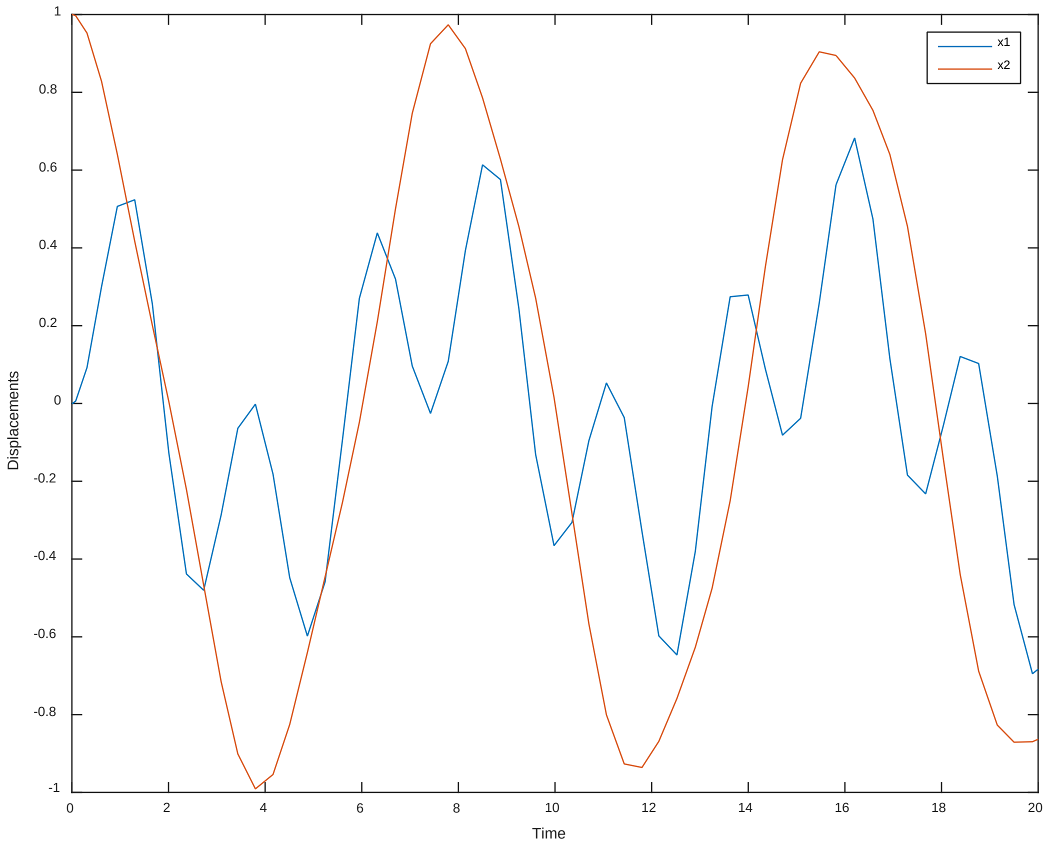

Figure 6- System Response of Example 2

It can be seen that solving for the response of a two degree of freedom system using Simulink only involves converting the system into state space form. Once the matrices are available, they can be inputted into the state space block and the response can be viewed. The system response can also be obtained using integrator blocks as done in example one where the equations are defined using integrators and summation blocks. That again is not much pain as doing Laplace transforms. The author prefers to use state space method because it results in a simplest Simulink model with just two blocks (figure 5).

Student Evaluation

A mid semester evaluation has been conducted to get an idea about how students feel about integrating MATLAB in lectures. Most of the students say that MATLAB/Simulink is a fun, interesting tool for simulating dynamic systems. Few of the comments are listed below:

- *MATLAB simulations provide a good visual representation of models and system response.*
- *I think MATLAB is a great way to show the behavior of system and how different parameters affect the system response*
- *MATALB simulations definitely show an easier way to solve problems and plotting system response really helps.*

Conclusion

An effort to improve students learning in dynamic systems and control course has been presented. Traditional approach of teaching dynamic systems could be boring for the students and they feel half of this course is just like another differential equations class. By integrating MATLAB simulations in lectures and make the students practice those in home works, the students realize the importance of the effect of different inputs and changing parameters on the system response. Overall, the author feels that these simulations provide an effective and enjoyable source of learning for the students. More feedback from the students will be collected at the end of semester on the overall teaching of the course.

References

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Dr. Ali is an Assistant Professor in the School of Engineering at Grand Valley State University since 2015. Prior to coming to Grand Valley, she worked as a lecturer at the University of Wisconsin Platteville and at the University of Wisconsin Milwaukee for two years. Dr. Ali received Ph.D. in Mechanical Engineering from the University of Wisconsin Milwaukee in 2013. Her areas of interest and expertise include Dynamics, Controls, Vibrations, Mathematical Optimization, Multilevel Algorithms and Game Theory.

Appendix

Solution of Example 2 using Laplace Transforms:

Using F=0, K_{s3} =0 and all damper constants equal zero, the equations of motion are reduced to:

$$m_1\ddot{x}_1 + (K_{s1} + K_{s2})x_1 - K_{s2}x_2 = 0 \tag{1}$$

$$m_2\ddot{x}_2 - K_{s2}x_1 + K_{s2}x_2 = 0 \tag{2}$$

Taking Laplace transform of equation 1:

$$m_1[s^2X_1(s) - sx_1(0) - \dot{x}_1(0)] + (K_{s1} + K_{s2})X_1(s) - K_{s2}X_2(s) = 0$$

Taking Laplace transform of equation 2:

$$m_2[s^2X_2(s) - sx_2(0) - \dot{x}_2(0)] - K_{s2}X_1(s) + K_{s2}X_2(s) = 0$$

Applying initial conditions and solving for X_1 and X_2:

$$X_1(s) = \frac{K_{s2}}{m_1s^2+K_{s1}+K_{s2}}X_2(s) \tag{3}$$

$$X_2(s) = \frac{K_{s2}X_1(s)+m_2s}{m_2s^2+K_{s2}} \tag{4}$$

Plug in X_1(s) from equation 3 into equation 4 and solve for X_2(s):

$$X_2(s) = \frac{m_2s(m_1s^2 + K_{s1} + K_{s2})}{(m_2s^2 + K_{s2})(m_1s^2 + K_{s1} + K_{s2}) - K_{s2}^2}$$

Similarly,

$$X_1(s) = \frac{m_2K_{s2}s}{(m_2s^2 + K_{s2})(m_1s^2 + K_{s1} + K_{s2}) - K_{s2}^2}$$

Plug in the data:

$$X_2(s) = \frac{8s^3+48s}{8s^4+56s^2+32} = \frac{s^3+6s}{(s^2+0.6277)(s^2+6.3723)} \tag{5}$$

$$X_1(s) = \frac{16s}{8s^4+56s^2+32} = \frac{2s}{(s^2+0.6277)(s^2+6.3723)} \tag{6}$$

Using partial fractions,

$$X_1(s) = \frac{0.7923C_1}{(s^2+0.6277)} + \frac{C_2s}{(s^2+0.6277)} + \frac{2.5243C_3}{(s^2+6.3723)} + \frac{C_4s}{(s^2+6.3723)} \tag{7}$$

$$X_2(s) = \frac{0.7923C_5}{(s^2+0.6277)} + \frac{C_6s}{(s^2+0.6277)} + \frac{2.5243C_7}{(s^2+6.3723)} + \frac{C_8s}{(s^2+6.3723)} \tag{8}$$

To solve for constants C_1 through C_4, equate equations 6 and 7. This will result in four equations (by equating the coefficients of **s** terms) in four unknowns which can be calculated as:

$$C_1 = 0, C_2 = 0.3481, C_3 = 0, C_4 = -0.3481$$

Equation 7 can now be written as:

$$X_1(s) = 0.3481\frac{s}{(s^2+0.6277)} - 0.3481\frac{s}{(s^2+6.3723)} \tag{9}$$

Similarly to solve for the constants C_5 through C_8, equate equations 5 and 8 and solve the resulting four equations for the four unknowns:

$$C_5 = 0, C_6 = 0.9352, C_7 = 0, C_8 = 0.0648$$

Equation 8 can now be written as:

$$X_2(s) = 0.9352\frac{s}{(s^2+0.6277)} + 0.0648\frac{s}{(s^2+6.3723)} \tag{10}$$

Taking the inverse Laplace of equations 9 and 10 gives:

$$x_1(t) = 0.3481\cos 0.7923t - 0.3481\cos 2.5243t$$

$$x_2(t) = 0.9352\cos 0.7923t + 0.0648\cos 2.5243t$$